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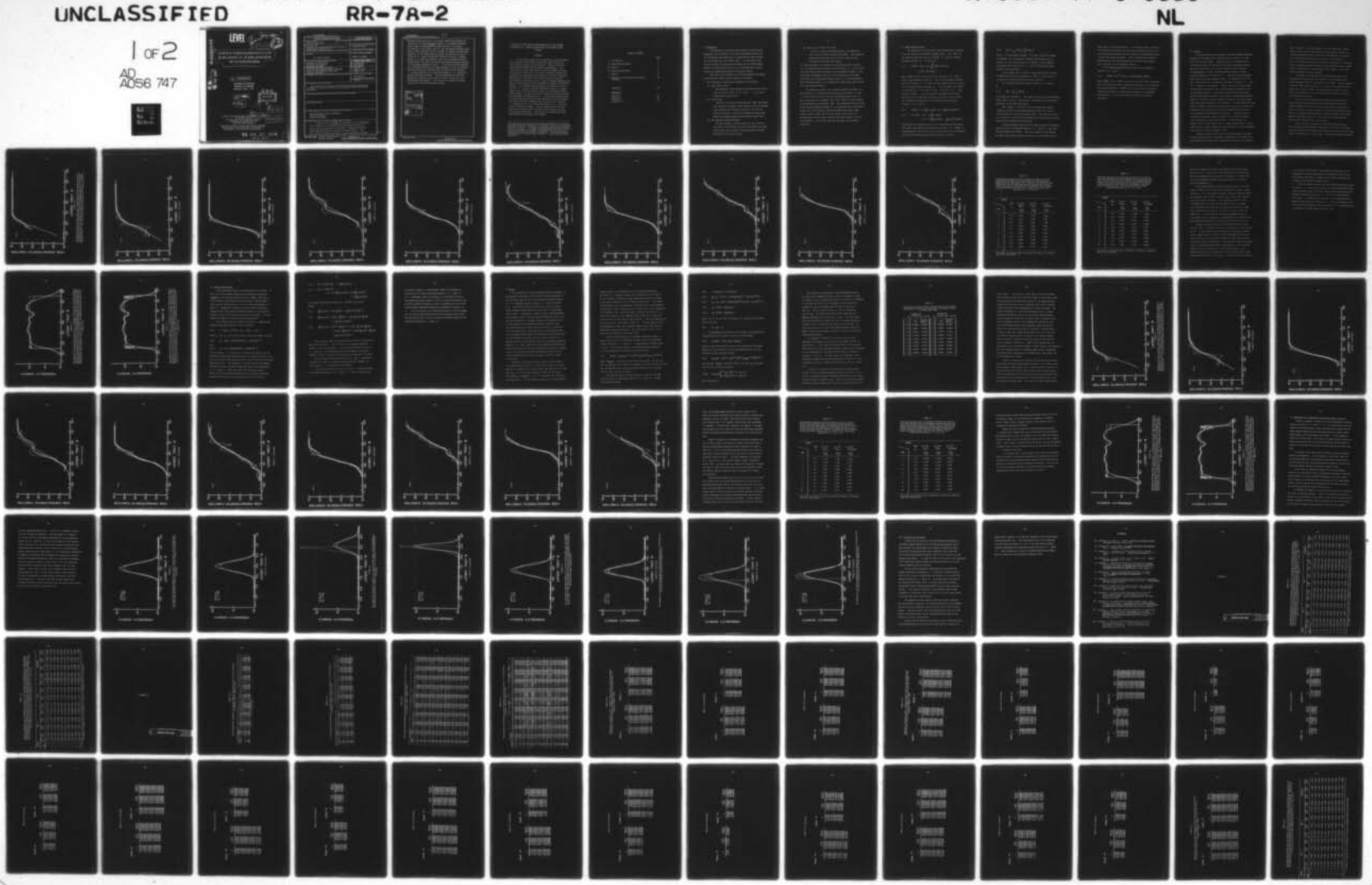

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terized by such facts that: 1) no prior mathematical models are assumed for the operating characteristics; 2) ~~we use~~ a relatively small number of examinees is used in the whole process; 3) the joint distribution of the latent trait θ and its maximum likelihood estimate is fully utilized, and 4) the method of moments is effectively applied. In the Normal Approach Method, the conditional distribution of θ , given its maximum likelihood estimate, is approximated by a normal distribution, with its two parameters derived theoretically, by using the estimated probability density function of the maximum likelihood estimate. In the Pearson System Method, the conditional distribution is approximated by one of the Pearson System distributions, depending upon the value of the criterion κ computed from the four conditional moments of θ , given its maximum likelihood estimate. The same simulated data for 500 hypothetical examinees and ten binary items are used, as in the Two-Parameter Beta Method. The results are compared with those obtained by the Two-Parameter Beta Method.

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ESTIMATION OF THE OPERATING CHARACTERISTICS OF ITEM RESPONSE CATEGORIES III: NORMAL APPROACH METHOD AND PEARSON SYSTEM METHOD

ABSTRACT

In the present study, two new variations of the Conditional P.D.F. Method for estimating the operating characteristics of item response categories are introduced. They are called the Normal Approach Method and the Pearson System Method. Just like the Two-Parameter Beta Method, these two methods are characterized by such facts that: 1) no prior mathematical models are assumed for the operating characteristics; 2) we use a relatively small number of examinees in the whole process; 3) the joint distribution of the latent trait θ and its maximum likelihood estimate is fully utilized, and 4) the method of moments is effectively applied. In the Normal Approach Method, the conditional distribution of θ, given its maximum likelihood estimate, is approximated by a normal distribution, with its two parameters derived theoretically, by using the estimated probability density function of the maximum likelihood estimate. In the Pearson System Method, the conditional distribution is approximated by one of the Pearson System distributions, depending upon the value of the criterion κ computed from the four conditional moments of θ, given its maximum likelihood estimate. The same simulated data for 500 hypothetical examinees and ten binary items are used, as in the Two-Parameter Beta Method. The results are compared with those obtained by the Two-Parameter Beta Method.

The research was conducted at the principal investigator's laboratory, 409 Austin Peay Hall, Department of Psychology, University of Tennessee, Knoxville, Tennessee. The group of graduate assistants was headed by Robert L. Trestman. The computer programming work was greatly assisted by one of the graduate assistants, Yeh Ching-Chuan, and some by the undergraduate assistant, Philip S. Livingston. The other assistants working for her at various times include C. I. Bonnie Chen and Li-Jen Jenny Chen.

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I Introduction

In estimating the operating characteristics of item response categories, such new methods as the Normal Approximation Method and the Two-Parameter Beta Method and its variations have been developed and tested (Samejima, 1977b, 1977d, 1978). In the present study, two more methods, the Normal Approach Method and the Pearson System Method, are presented and tried on a set of simulated data, and the results are compared with those of the previous studies.

Data used in this research are the same simulation data adopted in the previous studies, and are summarized as follows.

(1) The number of hypothetical examinees is five hundred.

(2) Their ability levels:

Each subgroup of five examinees are located at each ability level of -2.475, -2.425, -2.375, -2.325, ... , etc., up to 2.475 with the step of 0.05 .

(3) The Old Test:

This is a set of thirty five graded test items, each having four item score categories, which provides us with an approximately constant test information function, 21.63, for the interval of ability, [-3.0, 3.0], following the normal ogive model on the graded response level (Samejima, 1969, 1972).

(4) The examinees' ability estimates:

For each examinee, a response pattern on the old set of test items, or the Old Test, has been calibrated by the Monte Carlo method, and the maximum likelihood estimate has been obtained on this response pattern.

(5) The new set of binary test items:

This is a new set of ten binary items, or the New Test, each of which follows the normal ogive model. Each examinee's response pattern on this new set of test items has been calibrated by the Monte Carlo method.

The common procedure is to estimate the item characteristic function, i.e., the operating characteristic of the item score 1, of each of the ten binary items, without assuming any prior mathematical model. Once it has been accomplished, however, the two parameters in the normal ogive model on the dichotomous response level, i.e., the discrimination and difficulty parameters, are estimated by a simple least square method, as an additional information.

The asymptotic property of the maximum likelihood estimate, i.e., the normality of its conditional distribution, given ability θ, with θ itself and the inverse of the test information function $I(\theta)$ as the two parameters (cf. Samejima, 1975, 1977a, 1977b), is fully utilized. In the Two-Parameter Beta Method, $g(\hat{\theta})$, the probability density function of the maximum likelihood estimate $\hat{\theta}$, is approximated by a polynomial of degree 3 or 4 by the method of moments (Elderton and Johnson, 1969; Johnson and Kotz, 1970), and two cases, Degree 3 and 4 Cases, are distinguished from each other, depending upon which degree is used for the polynomial. We will deal with these two cases in the present study also.

II Normal Approach Method

The Conditional P.D.F. Method has been introduced as a variation of the Two-Parameter Beta Method (Samejima, 1978). Let $\phi(\theta|\hat{\theta})$ be the conditional density function of ability θ, given its maximum likelihood estimate $\hat{\theta}$. We can write

$$(2.1) \qquad \phi(\theta|\hat{\theta}) = \psi(\hat{\theta}|\theta)\ f(\theta)\ [\int_{-\infty}^{\infty} \psi(\hat{\theta}|\theta)\ f(\theta)\ d\theta]^{-1}$$

$$= \psi(\hat{\theta}|\theta)\ f(\theta)\ [g(\hat{\theta})]^{-1} ,$$

where $\psi(\hat{\theta}|\theta)$ is the conditional density function of $\hat{\theta}$, given θ, which is approximated by $n(\theta,\sigma^2)$, $f(\theta)$ is the probability density function of ability θ, and $g(\hat{\theta})$ is the probability density function of the maximum likelihood estimate $\hat{\theta}$. For the present set of data, or the Old Test, $\sigma^2 = (21.63)^{-1} \doteq 0.046$, and $f(\theta) = 0.2$. This function, $\phi(\theta|\hat{\theta})$, is not observable in the empirical situation. By virtue of the uniform test information function, however, we have the following relationships for the conditional expectation and variance of θ, given $\hat{\theta}$.

$$(2.2) \qquad E(\theta|\hat{\theta}) = \hat{\theta} + \sigma^2 \frac{d}{d\hat{\theta}} \log g(\hat{\theta}) = \hat{\theta} + \sigma^2[\frac{d}{d\hat{\theta}} g(\hat{\theta})][g(\hat{\theta})]^{-1} .$$

$$(2.3) \qquad Var.(\theta|\hat{\theta}) = \sigma^2[1 + \sigma^2 \frac{d^2}{d\hat{\theta}^2} \log g(\hat{\theta})]$$

$$= \sigma^2[1 + \sigma^2\{\frac{d^2}{d\hat{\theta}^2} g(\hat{\theta}) \cdot g(\hat{\theta}) - [\frac{d}{d\hat{\theta}} g(\hat{\theta})]^2\}\{g(\hat{\theta})\}^{-2}] .$$

Using these two conditional moments, in the Two-Parameter Beta Method, with $g(\hat{\theta})$ approximated by a polynomial of degree 3 or 4, $\phi(\theta|\hat{\theta})$ is approximated by a Beta density function with a priori set two parameters. Then the estimated item characteristic function is given by

$$(2.4)\qquad \hat{P}_g(\theta) = \sum_{s\varepsilon G} \hat{\phi}(\theta|\hat{\theta}_s)[\sum_{s=1}^{N} \hat{\phi}(\theta|\hat{\theta}_s)]^{-1} ,$$

where s is an individual examinee, N (= 500) is the total number of examinees, G is the group of examinees who answered item g correctly, or the "success" group, and $\hat{\phi}(\theta|\hat{\theta}_s)$ is the estimated conditional density function of θ, given the maximum likelihood estimate of the individual s.

In the present study, instead of using a Beta density function, a normal density function is adopted for $\hat{\phi}(\theta|\hat{\theta})$, and used in (2.4) to obtain the estimated item characteristic function.

The estimated probability density function of θ is given by

$$(2.5)\qquad \hat{f}(\theta) = \frac{1}{N} \sum_{s=1}^{N} \hat{\phi}(\theta|\hat{\theta}_s) ,$$

as an additional information. The simple least squares method (Samejima, 1977d, 1978) is used for estimating the discrimination and difficulty parameters in the normal ogive model.

When $\hat{\phi}(\theta|\hat{\theta}_s)$ is replaced by the true conditional density, $\phi(\theta|\hat{\theta}_s)$, in (2.4), the resulting function is called the criterion item characteristic function (Samejima, 1978). This function is used as a criterion in evaluating the estimated item characteristic functions obtained by the Conditional P.D.F. Method.

The Normal Approach Method has a convenient property of requiring only the first two conditional moments of θ, given $\hat{\theta}$, just like the Two-Parameter Beta Method. Since the estimation of the moments of higher order tends to be more inaccurate, it is desirable to use a

fewer number of conditional moments. In so doing, however, the variety of the shapes of the density function will be restricted by the use of too few conditional moments, and the estimation of the operating characteristics will become inaccurate. How to counterbalance these two opposing factors and produce optimal results is the crucial point, and is the focus of the researcher's effort.

In the Normal Approach Method, we have for the conditional density of θ , given $\hat{\theta}$,

(2.6) $$\hat{\phi}(\theta|\hat{\theta}) = [2\pi]^{-1/2}\exp[-\{\theta - E(\theta|\hat{\theta})\}^2/\{2\mathrm{Var.}(\theta|\hat{\theta})\}] ,$$

where $E(\theta|\hat{\theta})$ and $\mathrm{Var.}(\theta|\hat{\theta})$ are given by (2.2) and (2.3) respectively. Since this function is always unimodal and symmetric, it is conceivable that the approximation of $\phi(\theta|\hat{\theta})$ by (2.6) will cause a substantial distortion and result in an inaccurate estimation of the operating characteristics. We will wait and see the result to reach any conclusion.

III Results

As was mentioned earlier (Samejima, 1977d), the maximum likelihood estimate for the examinee No. 2 turned out to be -3.0555, i.e., much lower than the other values. Because of this, this examinee has to be excluded in Degree 4 Case, since the estimated probability density function at this value of $\hat{\theta}$ assumes a negative value, although the approximation to $g(\hat{\theta})$ by the polynomial of degree 3 provides a positive density at this point of $\hat{\theta}$. There are six more examinees whom we have to exclude in Degree 4 Case, i.e., Subjects 99, 101, 201 296, 299 and 300, since the estimated conditional variance of θ , given $\hat{\theta}$, turned out to be negative. Thus we could use the total number of 500 examinees in Degree 3 Case, and 493 examinees in Degree 4 Case, in the present research. We actually used 499 subjects in Degree 3 Case and 486 subjects in Degree 4 Case, however, excluding one subject in Degree 3 Case, and seven more subjects in Degree 4 Case, whose fourth conditional moments turned out to be negative. This exclusion reduces the number of subjects used in the present study in each case, but makes the comparison with the results of the Pearson System Method, which will be introduced in the following section, easier. Thus the examinees excluded in the present study are Subject 2 ($\theta = -2.425$) in Degree 3 Case, and Subjects 1, 101, 201 and 401 ($\theta = -2.475$), Subject 2 , Subject 3 ($\theta = -2.375$), Subject 4 ($\theta = -2.325$), Subject 296 ($\theta = 2.275$), Subject 98 ($\theta = 2.375$), Subjects 99, 199 and 299 ($\theta = 2.425$); and Subjects 300 and 500 ($\theta = 2.475$), in Degree 4 Case.

Figure 3-1 presents the estimated item characteristic functions of the ten binary items in both Degree 3 and 4 Cases, by a broken curve and a dotted curve respectively, along with the criterion item character-

istic functions. From these graphs, it is very clear that, in both Degree 3 and 4 Cases, the estimated item characteristic function in the Normal Approach Method is practically identical with the criterion item characteristic function for every item in the interval of θ [-2.0, 2.0], where the accuracy of estimation is important. This means that the present method has provided us with practically perfect results within the limitation of the Conditional P.D.F. Method, as far as the present simulation data are concerned.

It should be recalled that the practically same results were obtained by the Conditional P.D.F. Method of the Two-Parameter Beta Method (Samejima, 1978), in both Degree 3 and 4 Cases. We must conclude, therefore, that, without waiting for the results of the Pearson System Method, that these two methods have produced most successful results that the Conditional P.D.F. Method can possibly achieve, as far as the present data are concerned.

The fact that the Normal Approach Method has provided us with results as good as those produced by the Two-Parameter Beta Method somewhat puzzles us, however, since the normal density function does not have the kind of variety in shape as the Beta density function does. There is a strong possibility that the "too good" results by the Normal Approach Method are partly due to the nature of the simulation data, and this will be discussed in a later section.

Tables 3-1 and 3-2 present the estimated discrimination parameters and difficulty parameters respectively, for the ten binary items, using the range of the estimated item characteristic function, [0.05, 0.95], within the interval of θ , [-2.4, 2.4]. The same least square procedure

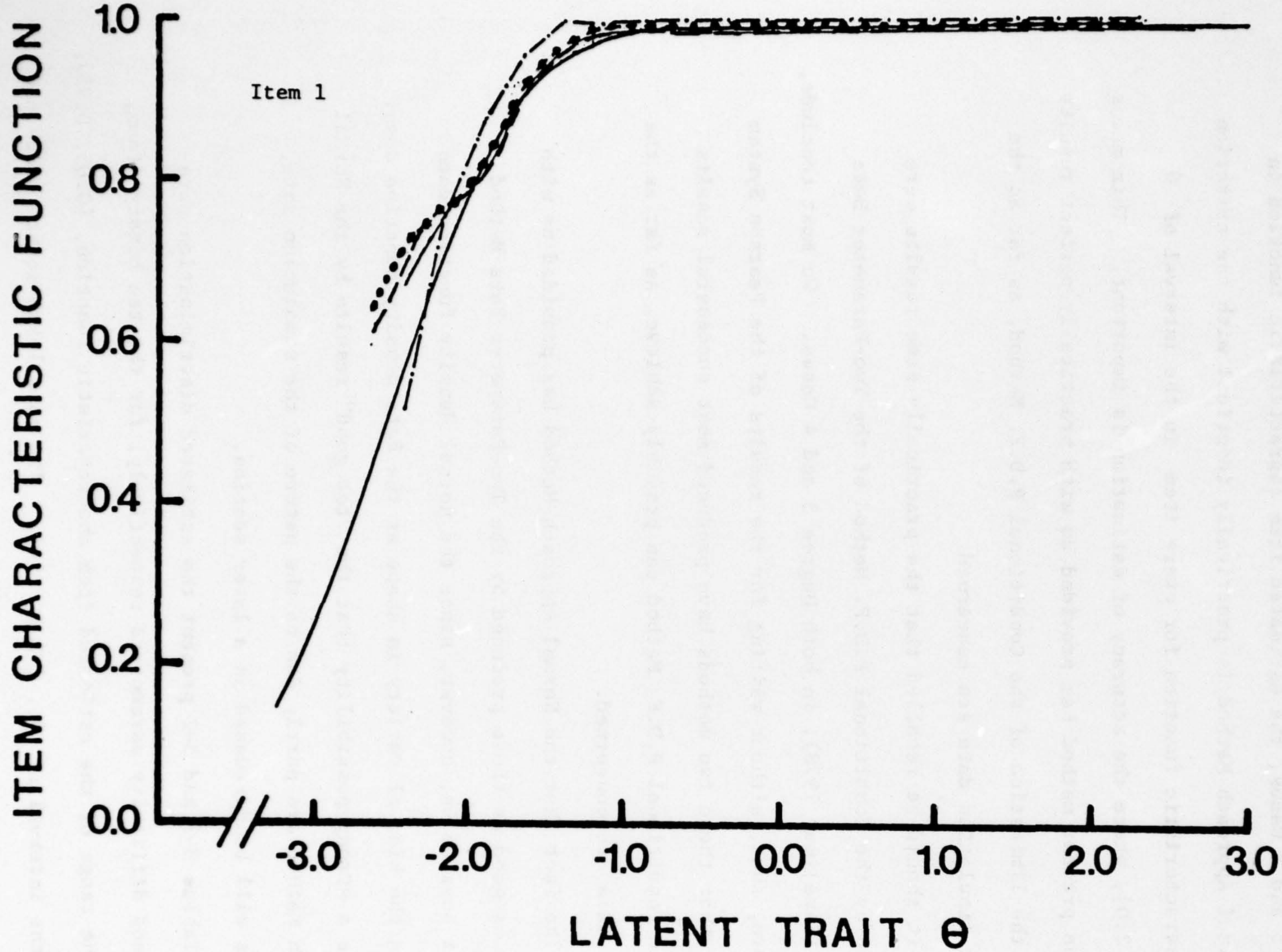
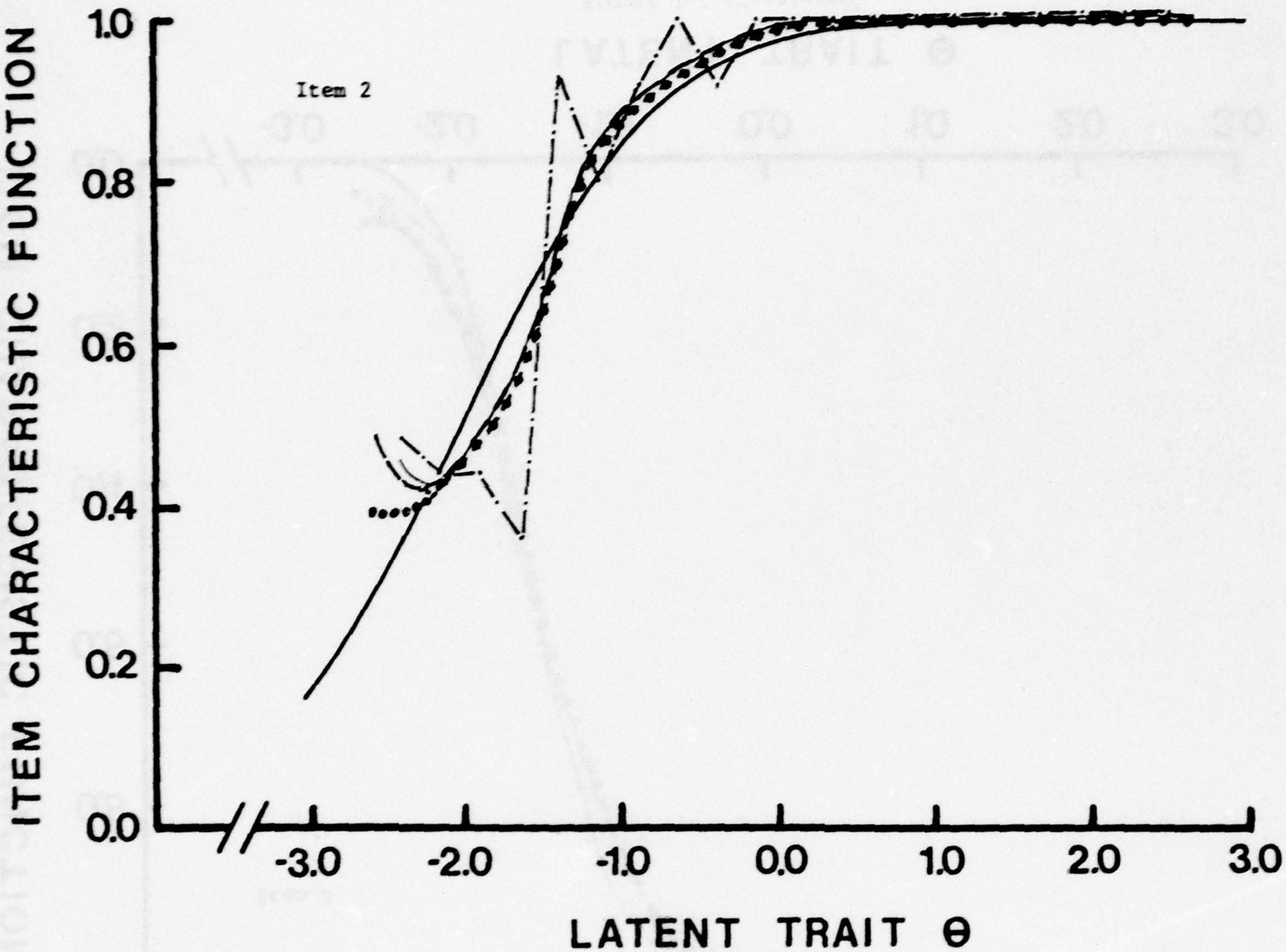


FIGURE 3-1

Estimated Item Characteristic Functions by the Normal Approach Method in Degree 3 Case (Broken Curve) and in Degree 4 Case (Dotted Curve), with the Criterion Item Characteristic Function (Thin Solid Curve), the Frequency Ratios of θ (Broken and Dotted Line) and the True Item Characteristic Function (Thick Solid Curve)



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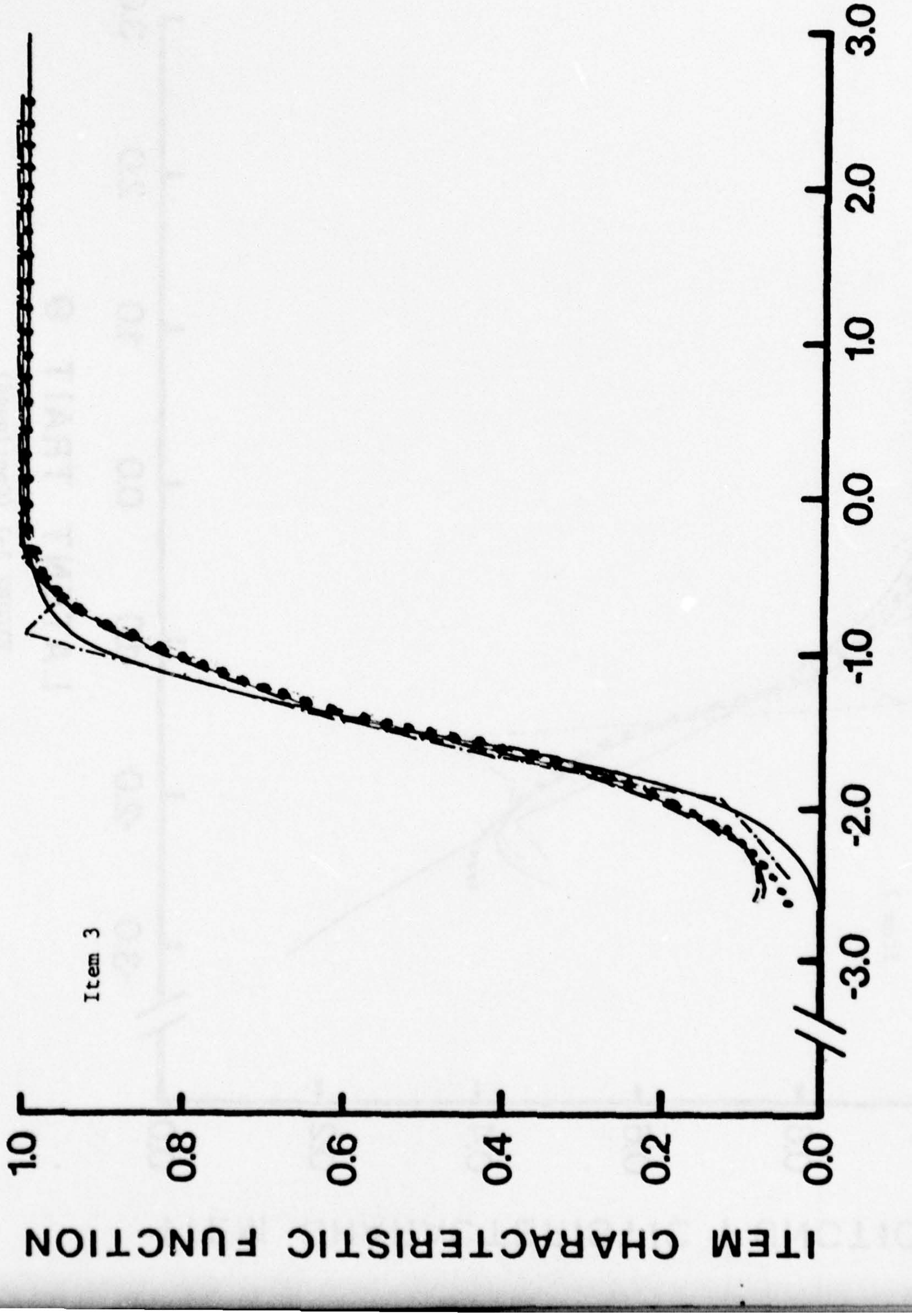

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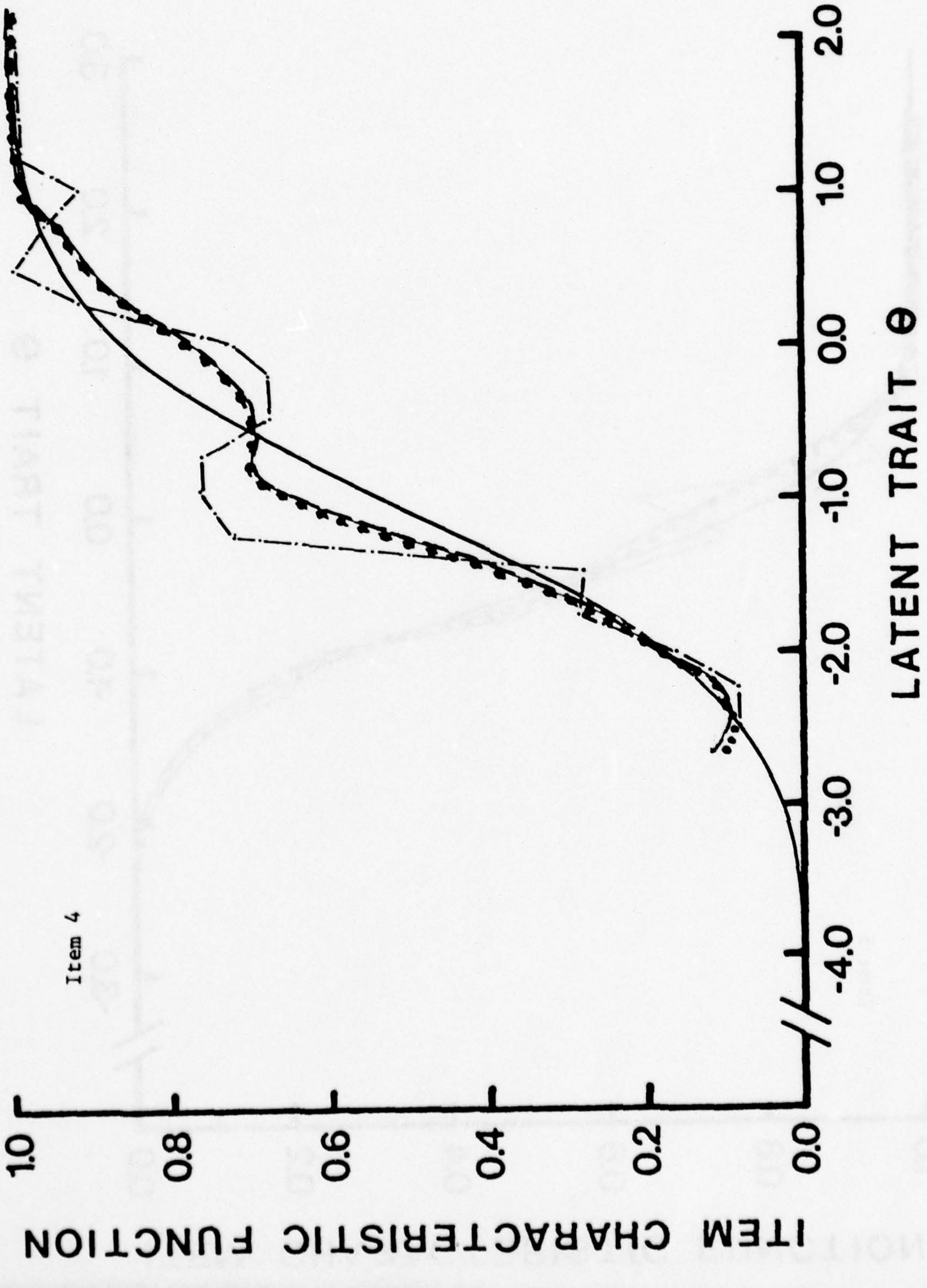

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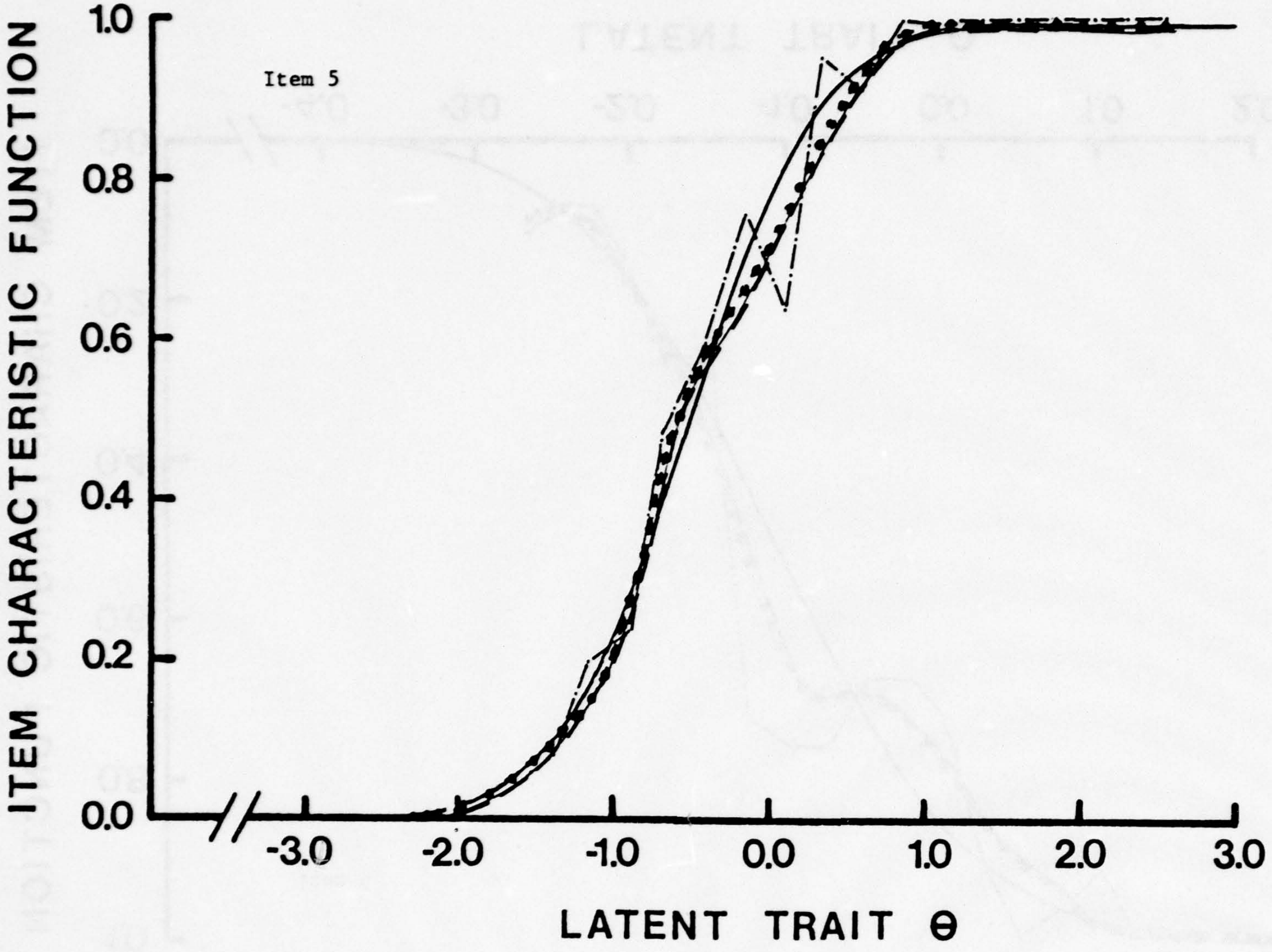

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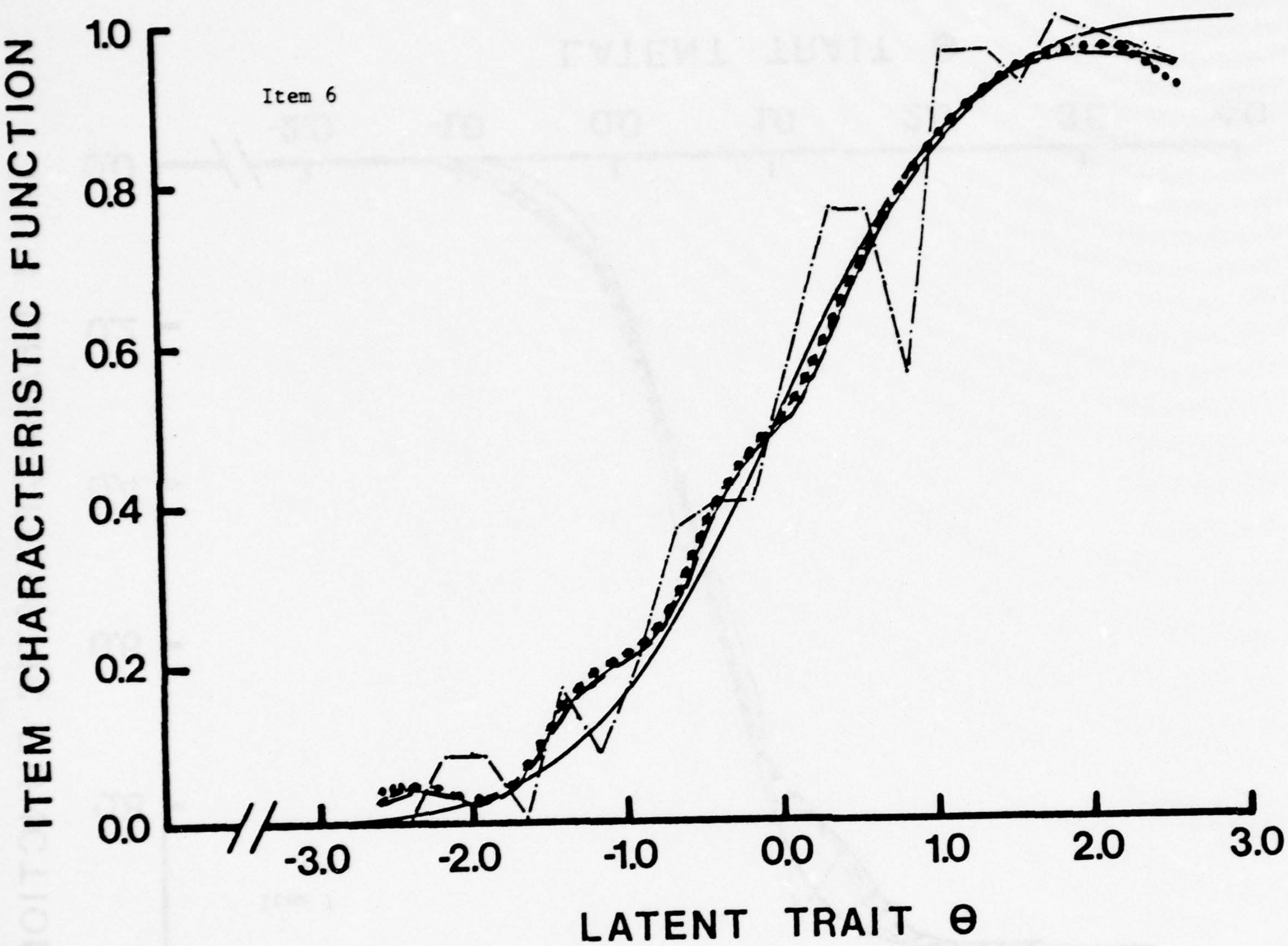

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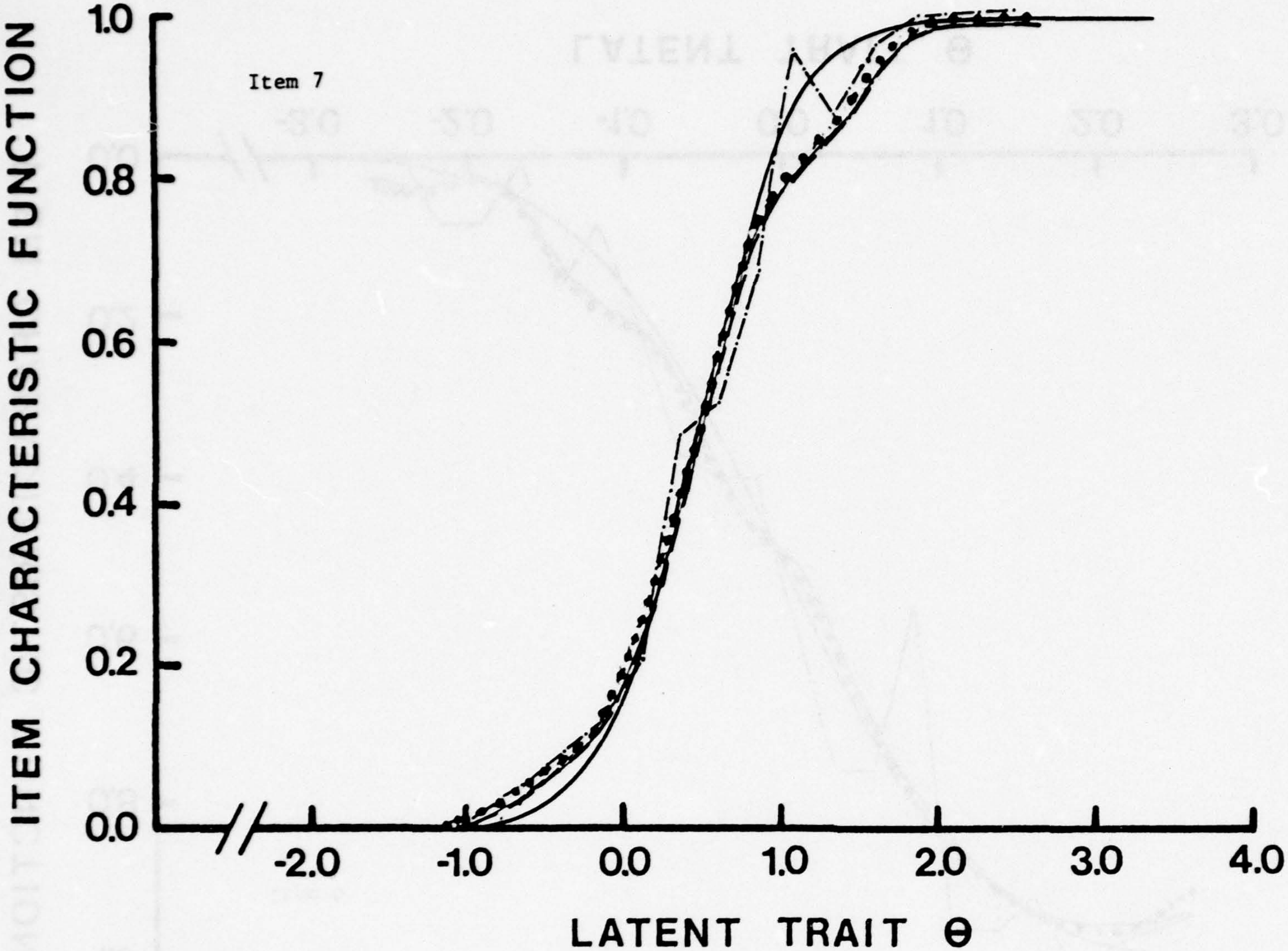

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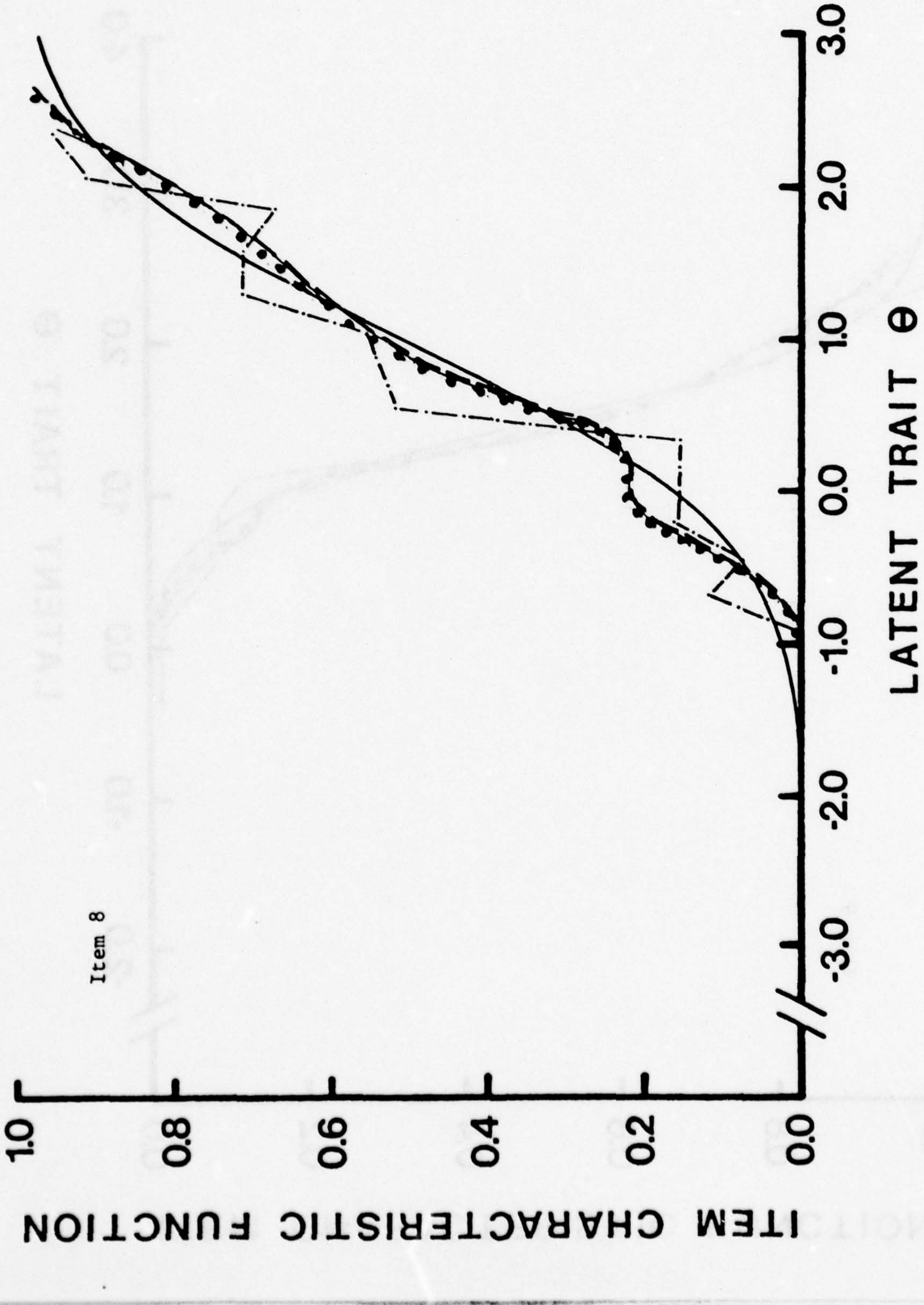

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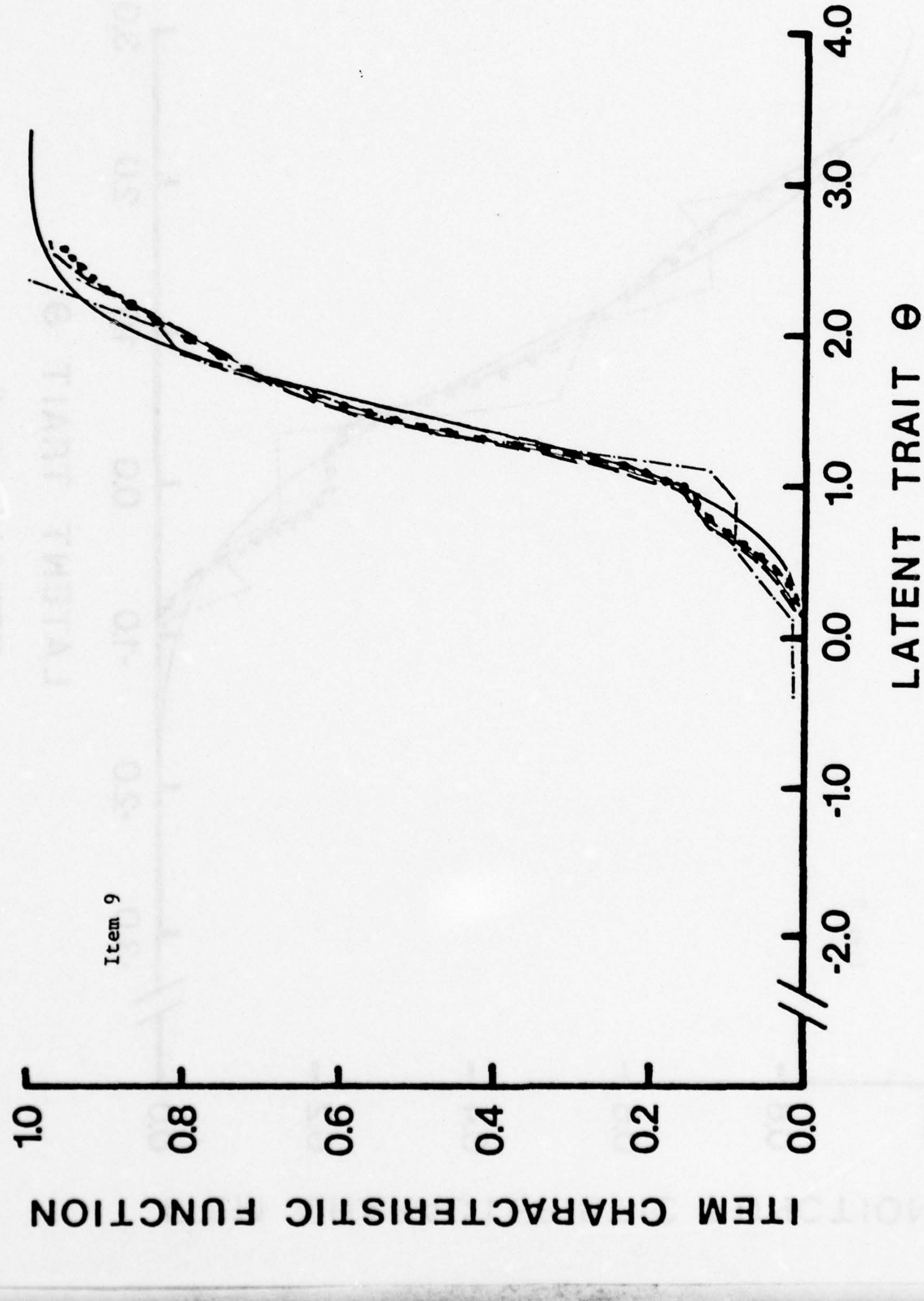

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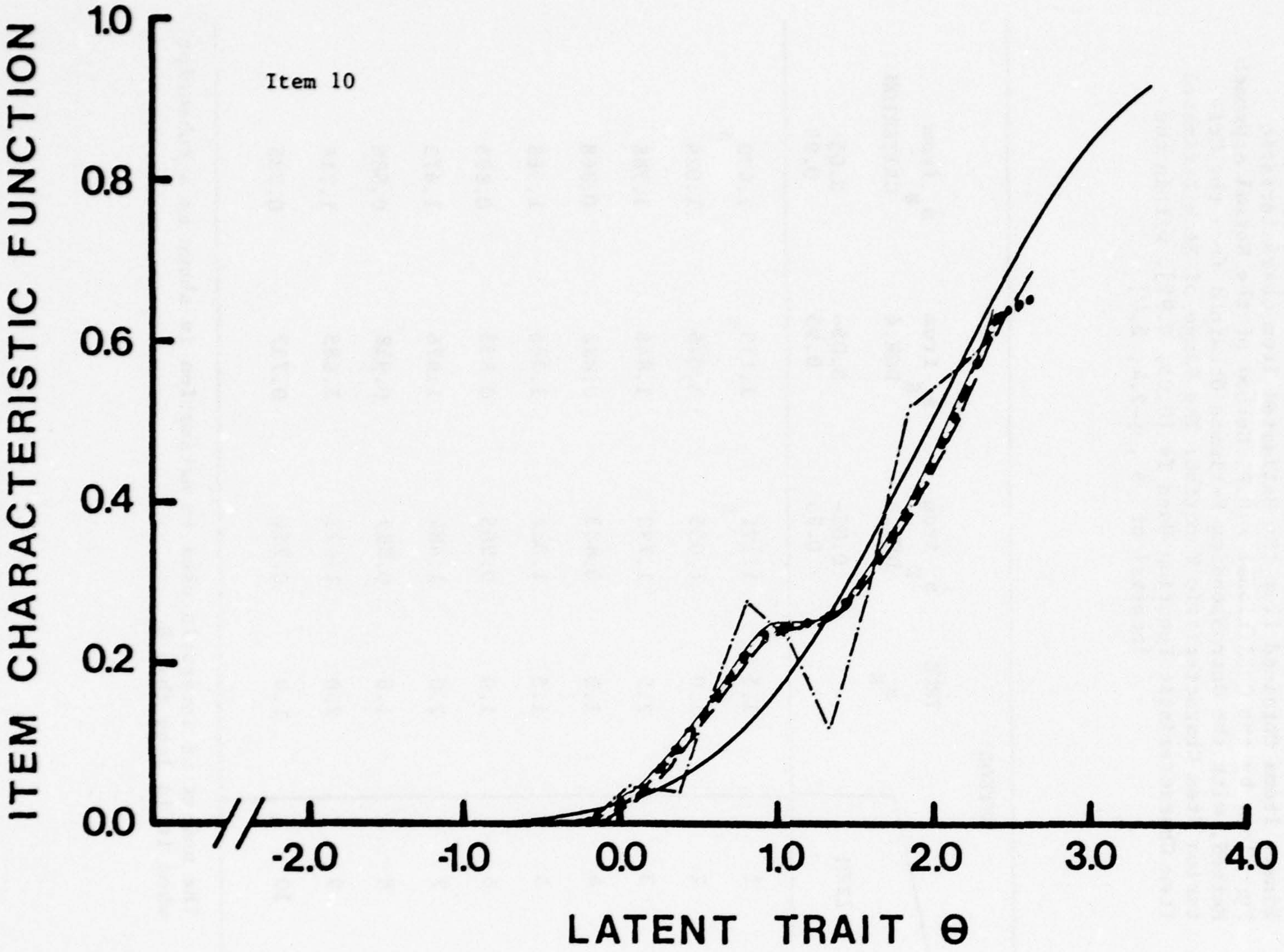


FIGURE 3-1 (Continued)

TABLE 3-1

Discrimination Parameter and Its Estimates of Each of the Ten Binary Items Obtained from the Estimated Item Characteristic Functions by the Conditional P.D.F. Method of the Normal Approach Method, with the Corresponding Estimate Obtained from the Criterion Item Characteristic Function: The Range of Each Estimated Item Characteristic Function Used Is [0.05, 0.95], within the Interval of θ, [-2.4, 2.4]

METHOD / ITEM	TRUE a_g	$\hat{a}_g$ from DGR.3 0.05-0.95	$\hat{a}_g$ from DGR.4 0.05-0.95	$\hat{a}_g$ from CRITERION 0.05-0.95
1	1.5	1.171_5	1.179_5	1.400_5
2	1.0	1.055	1.078	1.024
3	2.5	1.793	1.846	1.788
4	1.0	0.883	0.882	0.868
5	1.5	1.382	1.366	1.368
6	1.0	0.905	0.833	0.895
7	2.0	1.484	1.476	1.473
8	1.0	0.887	0.918	0.886
9	2.0	1.673	1.685	1.716
10	1.0	0.714	0.717	0.725

The number of intervals used in estimation is shown as a subscript when it is less than 6 .

TABLE 3-2

Difficulty Parameter and Its Estimates of Each of the Ten Binary Items Obtained from the Estimated Item Characteristic Functions by the Conditional P.D.F. Method of the Normal Approach Method, with the Corresponding Estimate Obtained from the Criterion Item Characteristic Function: The Range of Each Estimated Item Characteristic Function Used Is [0.05, 0.95], within the Interval of θ , [-2.4, 2.4]

METHOD / ITEM	TRUE b_g	$\hat{b}_g$ from DGR.3 0.05-0.95	$\hat{b}_g$ from DGR.4 0.05-0.95	$\hat{b}_g$ from CRITERION 0.05-0.95
1	-2.5	-2.792_5	-2.799_5	-2.651_5
2	-2.0	-1.966	-1.957	-2.002
3	-1.5	-1.494	-1.497	-1.507
4	-1.0	-0.990	-0.989	-1.005
5	-0.5	-0.468	-0.472	-0.472
6	0.0	-0.073	-0.048	-0.075
7	0.5	0.522	0.527	0.527
8	1.0	0.939	0.958	0.981
9	1.5	1.502	1.508	1.502
10	2.0	2.128	2.131	2.118

The number of intervals used in estimation is shown as a subscript when it is less than 6 .

was used by changing the range of the estimated item characteristic function to [0.01, 0.99], [0.10, 0.90] and [0.15, 0.85] respectively, and the corresponding results are given in Appendix I, as Table A-1-1 for the discrimination parameter a_g and as Table A-1-2 for the difficulty parameter b_g .

The estimated parameter values in Tables 3-1 and 3-2 are very close to those of the criterion item characteristic functions, except for item 1 , i.e., the easiest item with a relatively high discrimination parameter, and sometimes are better than those of the criterion item characteristic functions. The exceptional result for item 1 is obviously due to the fact that the main part of the estimated item characteristic function which is used for the parameter estimation is outside the interval of θ , [-2.0, 2.0], as we can see in Figure 3-1. It should also be noted that the present results are very similar to those obtained by the Conditional P.D.F. Method of the Two-Parameter Beta Method (cf. Samejima, 1978, Tables 6-1 and 6-2).

Figures 3-2 and 3-3 present the estimated probability density function $\hat{f}(\theta)$, in Degree 3 and 4 Cases respectively, by heavy dotted curves. In these figures, also presented are corresponding results by the Conditional P.D.F. Method of the Two-Parameter Beta Method by broken curves, those from the true $\phi(\theta|\hat{\theta})$ by solid curves, and the theoretical $f(\theta)$ by dashed lines. Since the total number of subjects used in the Normal Approach Method is 499 in Degree 3 Case and 486 in Degree 4 Case, the slanted area in the uniform density, $f(\theta)$, should be discarded in evaluating the closeness of the result of the Normal Approach Method, in each figure. For an additional information,

in each graph, slanted lines in the opposite direction are used to indicated the area to be discarded in evaluating the result of the Two-Parameter Beta Method, to make the total number of subjects 499 in Degree 3 Case and 493 in Degree 4 Case.

These two figures indicate that the estimated probability density function of θ is very close to that of the Two-Parameter Beta Method, and is reasonably close to both the theoretical and "semi-theoretical" functions within the interval of θ , (-2.0, 2.0), in both Degree 3 and 4 Cases. It should be noted that the fit is slightly better in Degree 4 Case outside this interval of θ , which is anticipated from the fact that the probability density function of $\hat{\theta}$ is approximated by a polynomial of a higher degree in Degree 4 Case.

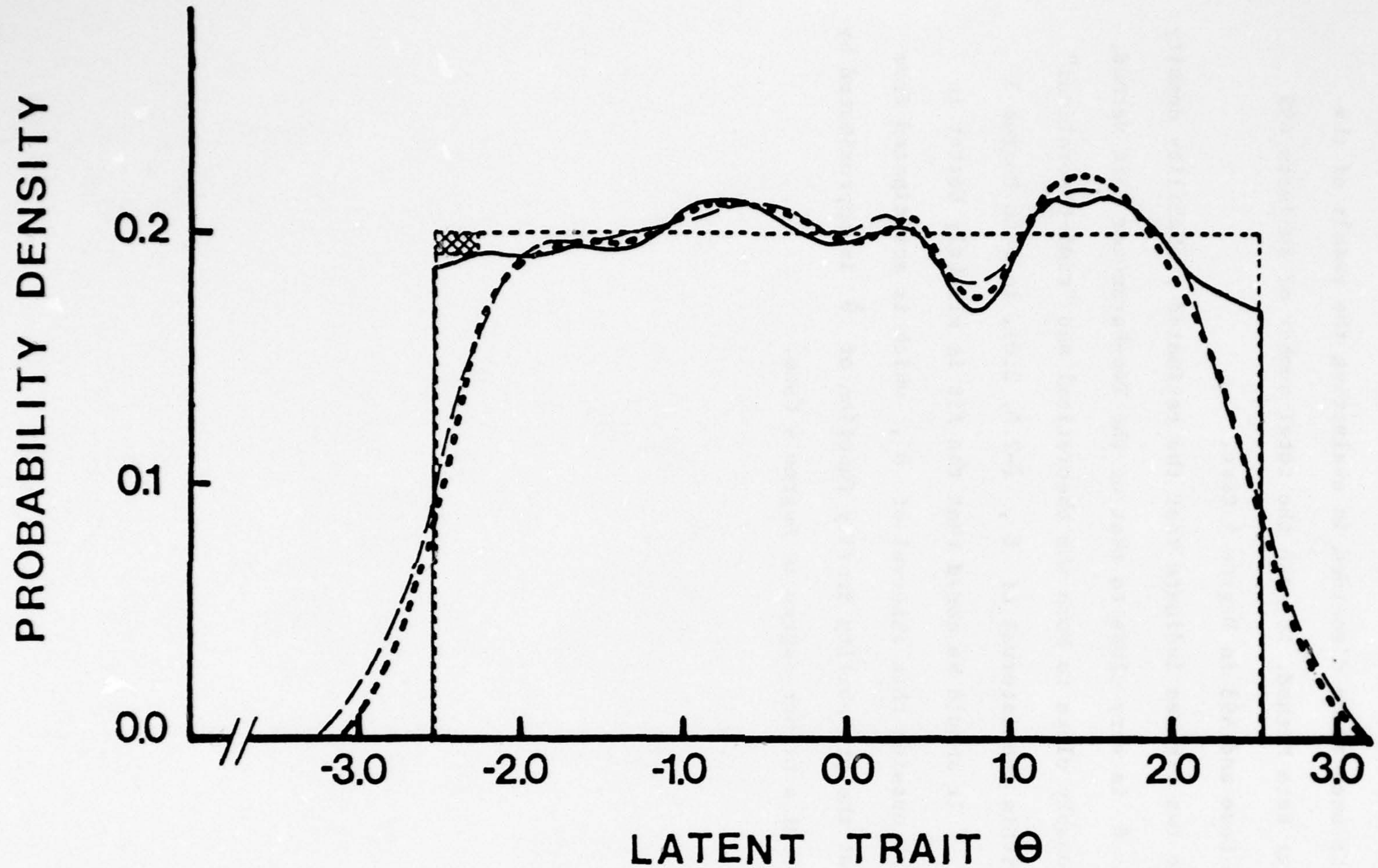


FIGURE 3-2

Approximated Marginal Density Function of θ Obtained from the 499 Conditional Density Functions of θ, Given Its Maximum Likelihood Estimate, in Degree 3 Case of the Normal Approach Method(Dashed Curve), with the Corresponding Result by the Two-Parameter Beta Method(Broken Curve), Along with the Density Function Obtained from the 500 True Conditional Density Functions(Solid Curve), and $f(\theta)$, the Theoretical Density Function of θ (Dashed Line)

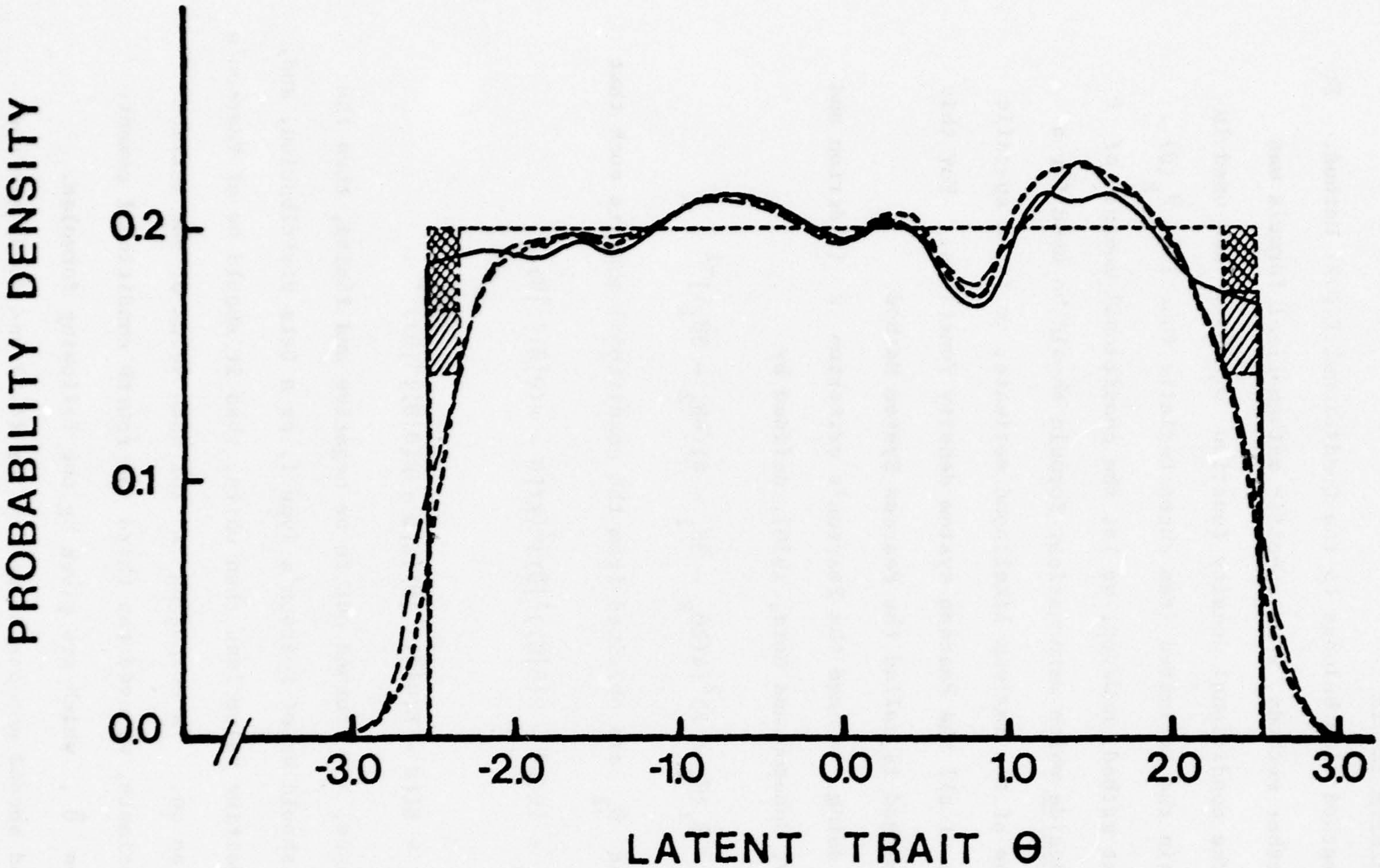


FIGURE 3-3

Approximated Marginal Density Function of θ Obtained from the 486 Conditional Density Functions of θ, Given Its Maximum Likelihood Estimate, in Degree 4 Case of the Normal Approach Method (Dashed Curve), with the Corresponding Result by the Two-Parameter Beta Method (Broken Curve), Along with the Density Function Obtained from the 500 True Conditional Density Functions (Solid Curve), and $f(\theta)$, the Theoretical Density Function of θ (Dashed Line)

IV <u>Pearson System Method</u>

This method also belongs to the Conditional P.D.F. Method. So far, in the other methods, some specific mathematical formula was <u>assumed</u> for the conditional density function $\phi(\theta|\hat{\theta})$, and used in (2.4) to obtain the estimated item characteristic function $\hat{P}_g(\theta)$. In the present method, however, we let the conditional moments of θ given $\hat{\theta}$, <u>decide</u> which mathematical formula should be used for a specific value of the maximum likelihood estimate, or for a specific examinee, out of all the Pearson system density functions. For this reason, the method is called the Pearson System Method.

In so doing, we need the Pearson's criterion κ (Elderton and Johnson, 1969; Johnson and Kotz, 1970), defined by

$$(4.1) \qquad \kappa = \beta_1(\beta_2 + 3)^2[4(2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)]^{-1} ,$$

where β_1 and β_2 are obtained from the conditional moments such that

$$(4.2) \qquad \beta_1 = \{E([\theta - E(\theta|\hat{\theta})]^3|\hat{\theta})\}^2\{E([\theta - E(\theta|\hat{\theta})]^2|\hat{\theta})\}^{-3}$$

and

$$(4.3) \qquad \beta_2 = E([\theta - E(\theta|\hat{\theta})]^4|\hat{\theta})\{E[\theta - E(\theta|\hat{\theta})]^2|\hat{\theta})\}^{-2} .$$

If, for instance, κ turned out to be negative and finite, then the distribution should be of Pearson's Type I, or a Beta distribution, and, if it is a positive value less than unity, then it should be of Pearson's Type IV, and so on. In computing κ for each value of the maximum likelihood estimate, we need the third and fourth conditional moments of θ , given $\hat{\theta}$, which are given by the following formulae. (The first and second moments necessary for (4.2) and (4.3) are the conditional expectation and variance given as (2.2) and (2.3).)

$$(4.4)\qquad E[\{\theta - E(\theta|\hat{\theta})\}^3|\hat{\theta}] = -\sigma^6[\frac{d^3}{d\theta^3}\log g(\hat{\theta})] \ .$$

$$(4.5)\qquad E[\{\theta - E(\theta|\hat{\theta})\}^4|\hat{\theta}]$$

$$= \sigma^4[3 + 6\sigma^2\{\frac{d^2}{d\theta^2}\log g(\hat{\theta})\} + 3\sigma^4\{\frac{d^2}{d\theta^2}\log g(\hat{\theta})\}^2$$

$$+ \sigma^4\{\frac{d^4}{d\theta^4}\log g(\hat{\theta})\}] \ .$$

The second through fourth derivatives of $\log g(\hat{\theta})$ are given as follows.

$$(4.6)\qquad \frac{d^2}{d\theta^2}\log g(\hat{\theta}) = [g(\hat{\theta})\cdot\frac{d^2}{d\theta^2}g(\hat{\theta}) - \{\frac{d}{d\theta}g(\hat{\theta})\}^2][g(\hat{\theta})]^{-2} \ .$$

$$(4.7)\qquad \frac{d^3}{d\theta^3}\log g(\hat{\theta}) = [\{g(\hat{\theta})\}^2\cdot\frac{d^3}{d\theta^3}g(\hat{\theta}) - 3g(\hat{\theta})\cdot\frac{d}{d\theta}g(\hat{\theta})\cdot\frac{d^2}{d\theta^2}g(\hat{\theta})$$

$$+ 2\{\frac{d}{d\theta}g(\hat{\theta})\}^3][g(\hat{\theta})]^{-3} \ .$$

$$(4.8)\qquad \frac{d^4}{d\theta^4}\log g(\hat{\theta}) = [\{g(\hat{\theta})\}^3\cdot\frac{d^4}{d\theta^4}g(\hat{\theta}) - 4\{g(\hat{\theta})\}^2\cdot\frac{d}{d\theta}g(\hat{\theta})\cdot\frac{d^3}{d\theta^3}g(\hat{\theta})$$

$$- 3\{g(\hat{\theta})\}^2\{\frac{d^2}{d\theta^2}g(\hat{\theta})\}^2 + 12g(\hat{\theta})\{\frac{d}{d\theta}g(\hat{\theta})\}^2\cdot\frac{d^2}{d\theta^2}g(\hat{\theta})$$

$$- 6\{\frac{d}{d\theta}g(\hat{\theta})\}^4][g(\hat{\theta})]^{-4} \ .$$

After a specific type of function has been selected according to the value of the criterion κ for each value of the maximum likelihood estimate , or for each examinee s , the conditional density function of θ , given $\hat{\theta}_s$, is estimated by the method of moments, and used as $\hat{\phi}(\theta|\hat{\theta}_s)$ in (2.4), to obtain the estimated item characteristic function.

The estimated probability density function, $\hat{f}(\theta)$, is obtained through (2.5), using the estimated conditional density $\hat{\phi}(\theta|\hat{\theta}_s)$ for each examinee s , as an additional information.

As we can see in (4.1), Pearson's criterion κ requires the first four conditional moments of θ , given $\hat{\theta}$. This fact implies that

the method is useless, or even harmful, unless the estimation is accurate up to the fourth conditional moments of θ , given $\hat{\theta}$. It is interesting to find out, therefore, if the method works with the present data, which consist of only five hundred observations, and in which the method of moments is used for graduating the set of five hundred maximum likelihood estimates by a polynomial of degree 3 or 4 . If the estimation of the conditional moments is accurate enough, then the Pearson System Method should have advantage over the Normal Approach Method, since it allows a variety of different shapes for the conditional density of θ , given $\hat{\theta}$.

V Results

The classifiction of the 500 maximum likelihood estimates $\hat{\theta}$ with respect to the types of their conditional distribution of θ has been made, using the first four estimated conditional moments of θ in computing Pearson's criterion κ, which is given by (4.1). The detailed results are shown in an earlier research report (Samejima, 1977d, Tables A-1-1 and A-1-2), in both Degree 3 and 4 Cases. To summarize, in Degree 3 Case, 318 distributions are of Pearson's Type I, 181 are normal distributions, and for one case the distribution is undefined because of the negative value of the estimated fourth conditional moment; in Degree 4 Case, 432 distributions are of Type I and 54 are of Type II, and for 13 cases the distribution is undefined because of the negative value of the estimated fourth moment, and sometimes of the negative value of the estimated second moment as well, and for one case it is undefined because of the negative value of the estimated probability density of $\hat{\theta}$. Thus, except for the one undefined distribution in Degree 3 Case and for the fourteen undefined distributions in Degree 4 Case, all the other conditional distributions turned out to be either Beta distributions (Pearson's Types I and II) or normal distributions.

The above results were obtained in such a way that Type I is assigned if the criterion κ is less than, or equal to, -0.001, Type II is assigned if it is greater than -0.001 and less than 0.001 and $(\beta_2 - 3)$ is less than or equal to -0.001, and the normal distribution is assigned if $(\beta_2 - 3)$ is greater than -0.001 and less than 0.001 . If we increase the range of error from ± 0.001 to

±0.025, however, the configurations of the frequencies change substantially, i.e., in Degree 3 Case, only 6 distributions are of Type I, 10 are of Type II, and 483 are normal distributions, whereas in Degree 4 Case these frequencies are 41, 41 and 404 respectively. This rather radical change in the configurations of different types of distributions in both Degree 3 and 4 Cases takes us as surprise, but a close examination of the values of β_2 reveals that many of them are very close to 3, like 2.999, 2.998, etc. (cf. Samejima, 1977d, Tables A-1-1 and A-1-2), and also there are many values of the criterion κ which are negative, but very close to zero, like -0.001, -0.002, etc. For this reason, it will be meaningless to treat them as Pearson's Type I cases, since they are practically normal distributions. The second range of error, ±0.025, therefore, was adopted in the present study.

Thus for the 483 values of the maximum likelihood estimate in Degree 3 Case and for the 404 values in Degree 4 Case the conditional density function of θ, given $\hat{\theta}$, is approximated by $\hat{\phi}(\theta|\hat{\theta})$ given by (2.6). For the other values of the maximum likelihood estimate it is approximated by a Beta density function such that

$$(5.1)\qquad \hat{\phi}(\theta|\hat{\theta}) = [B(p_{\hat{\theta}},q_{\hat{\theta}})]^{-1}\,(\theta-a_{\hat{\theta}})^{p_{\hat{\theta}}-1}(b_{\hat{\theta}}-\theta)^{q_{\hat{\theta}}-1}(b_{\hat{\theta}}-a_{\hat{\theta}})^{-(p_{\hat{\theta}}+q_{\hat{\theta}}-1)}\ ,$$

where $B(p_{\hat{\theta}},q_{\hat{\theta}})$ is the Beta function with the parameters $p_{\hat{\theta}}$ and $q_{\hat{\theta}}$, and $a_{\hat{\theta}}$ and $b_{\hat{\theta}}$ are the other two parameters of the Beta density function. This includes both Type I and Type II density functions with $p_{\hat{\theta}} \neq q_{\hat{\theta}}$ for Type I and $p_{\hat{\theta}} = q_{\hat{\theta}}$ for Type II.

The four parameters of the Beta density function are obtained from the first four conditional moments of θ, given $\hat{\theta}$, through the following formulae.

(5.2) $$r = 6(\beta_2-\beta_1-1) / (6+3\beta_1-2\beta_2) .$$

(5.3) $$p_{\hat\theta},\ q_{\hat\theta} = (r/2)[1 \pm (r+2)\{\beta_1[\beta_1(r+2)^2 + 16(r+1)]^{-1}\}^{1/2}] .$$

(5.4) $$b_{\hat\theta} - a_{\hat\theta} = \{E[(\theta - E[\theta|\hat\theta])^2|\hat\theta]\}^{1/2}\{\beta_1(r+2)^2 + 16(r+1)\}^{1/2} /2 .$$

(5.5) $$a_{\hat\theta} = E[\theta|\hat\theta] - p_{\hat\theta}(b_{\hat\theta}-a_{\hat\theta})/r .$$

(5.6) $$b_{\hat\theta} = E[\theta|\hat\theta] + q_{\hat\theta}(b_{\hat\theta}-a_{\hat\theta})/r .$$

Since $\beta_1 = 0$ for the Type II category, it is easily seen from (5.3) that in this case

(5.7) $$p_{\hat\theta} = q_{\hat\theta} = r/2 .$$

In programming this process for the computer, the subroutine for the gamma function was used by virtue of the relationship

(5.8) $$B(p_{\hat\theta},q_{\hat\theta}) = \Gamma(p_{\hat\theta})\ \Gamma(q_{\hat\theta})\ [\Gamma(p_{\hat\theta}+q_{\hat\theta})]^{-1} .$$

Because of the restriction of the size of the parameter of the gamma function for the subroutine, the following approximation, which is obtained from the Stirling's Formula,

(5.9) $$B(p_{\hat\theta},q_{\hat\theta}) \doteq (2\pi)^{1/2}\ [p_{\hat\theta}^{\ 2p_{\hat\theta}-1}\ q_{\hat\theta}^{\ 2q_{\hat\theta}-1}\ (p_{\hat\theta}+q_{\hat\theta})^{1-2(p_{\hat\theta}+q_{\hat\theta})}]^{1/2} ,$$

was used when $(p_{\hat\theta}+q_{\hat\theta})$ exceeds 57 and both $p_{\hat\theta}$ and $q_{\hat\theta}$ are greater than 15, and, in similar situations,

(5.10) $$B(p_{\hat\theta},q_{\hat\theta}) \begin{cases} \doteq \Gamma(q_{\hat\theta})\ p_{\hat\theta}^{-q_{\hat\theta}} & \text{for } q_{\hat\theta} \leq 15 , \\ \doteq \Gamma(p_{\hat\theta})\ q_{\hat\theta}^{-p_{\hat\theta}} & \text{for } p_{\hat\theta} \leq 15 , \end{cases}$$

were used, however.

The estimated four parameters of the Beta density function for each of the 6 examinees of Type I and the 10 examinees of Type II in Degree 3 Case is presented in Tables A-2-1 and A-2-2 of Appendix II, and those for each of the 41 examinees of Type I and the 41 examinees of Type II in Degree 4 Case is shown in Tables A-2-3 and A-2-4. We notice that in Table A-2-3 the two parameters, $p_{\hat{\theta}}$ and $q_{\hat{\theta}}$, for Subjects 100, 104 and 198 turned out to be negative, although they are very close to zero. For this reason, these three subjects are excluded from the total group of subjects in Degree 4 Case, to make the total number of subjects 483, i.e., three less than the one in the Normal Approach Method, whereas in Degree 3 Case the total number of subjects is 499, as it was in the Normal Approach Method.

It is apparent in Tables A-2-2 and A-2-4 that for these examinees the parameters $p_{\hat{\theta}}$ and $q_{\hat{\theta}}$ are not really one half of the value of r, and in some cases the difference is substantial. This fact is due to the relatively "generous" amount of error permitted in the categorization of the subjects with respect to Pearson's criterion κ and β_2. For this reason, the computation of the conditional density was performed through (5.1), just as it was for the subjects of Type I, without recalculating $p_{\hat{\theta}}$ and $q_{\hat{\theta}}$ through (5.7).

To check the accuracy of the approximation using Stirling's Formula, (5.1) was used by replacing the Beta function by (5.9) for all the six subjects of Type I in Degree 3 Case and the thirty eight subjects of Type I in Degree 4 Case. Table 5-1 presents two examples from these results, whose parameter values are relatively small, but

TABLE 5-1

Comparison of Pearson's Type I Density Function with the One Approximated by Stirling's Formula When the Parameter Values Are Relatively Small

Subject 101 $p_{\hat{\theta}} = 11.9187$ $q_{\hat{\theta}} = 8.1981$			Subject 392 $p_{\hat{\theta}} = 10.2720$ $q_{\hat{\theta}} = 15.5651$		
θ	True	Approximation	θ	True	Approximation
-3.6	0.00000	0.00000	1.6	0.00009	0.00010
-3.4	0.00137	0.00139	1.8	0.06382	0.06448
-3.2	0.06110	0.06190	2.0	0.76473	0.77261
-3.0	0.51138	0.51808	2.2	1.86010	1.87927
-2.8	1.52044	1.54037	2.4	1.62962	1.64642
-2.6	1.90820	1.93320	2.6	0.59455	0.60067
-2.4	0.90790	0.91979	2.8	0.08366	0.08453
-2.2	0.08937	0.09054	3.0	0.00324	0.00327
-2.0	0.00006	0.00006	3.2	0.00001	0.00001

not too small. As we can see in this table, even with relatively small parameter values like 11.9187 and 8.1981 or 10.2720 and 15.5651 the approximation using Stirling's Formula for the gamma functions through (5.9) provides us with considerably good results, although there is no need to use the approximation in cases like them. For further information as to how the approximation works, a similar comparison was made for each of the six subjects, whose parameter values are given in Table A-2-1, of Pearson Type I in Degree 3 Case and is presented in Table A-2-5 in Appendix II, and also for each of the thirty four subjects, whose parameter values are given in Table A-2-2, the sum of which does not exceed 57, of Pearson Type I in Degree 4 Case, and is presented as Table A-2-6, and for each of two subjects of Pearson Type II in Degree 4 Case, whose parameter values are given in Table A-2-4, the sum of which is less than 57, and is presented as Table A-2-7. These results indicate that the approximation is not only good enough for larger values of the parameters, but also reasonably good for considerably small values, although for some subjects whose parameters are extremely small, like Subject 397, the discrepancies are larger.

Figure 5-1 presents the estimated item characteristic functions obtained by the Pearson System Method for Degree 3 Case (broken curve) and for Degree 4 Case (dotted curve), together with the criterion item characteristic function (thin, solid curve) and the frequency ratio (broken and dotted curve) of the "success" group of individuals to the total group for each subinterval of θ with the width of 0.25 , for each of the ten binary items. The results are almost identical with

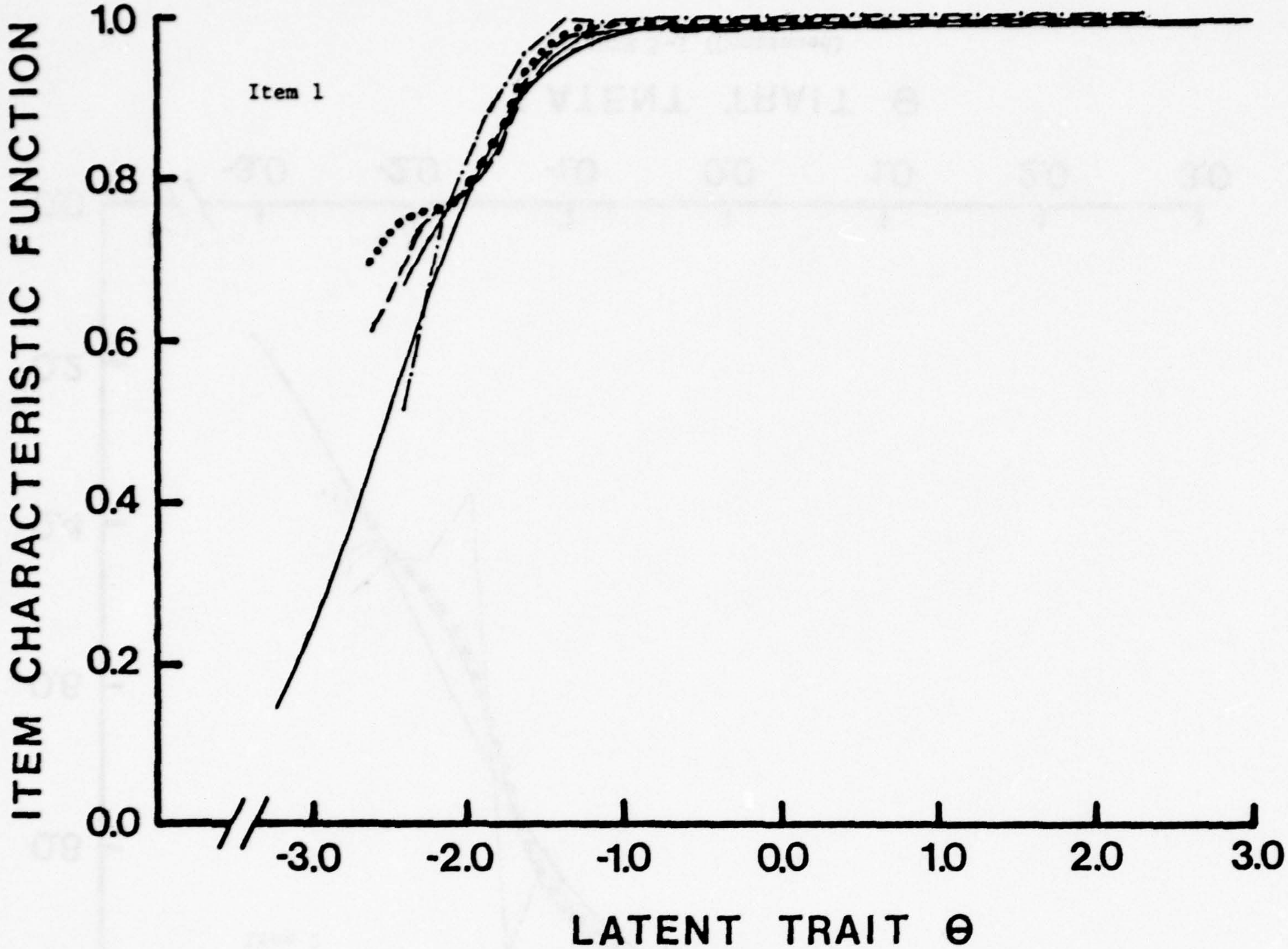


FIGURE 5-1

Estimated Item Characteristic Functions by the Pearson System Method in Degree 3 Case (Broken Curve) and in Degree 4 Case (Dotted Curve), with the Criterion Item Characteristic Function (Thin Solid Curve), the Frequency Ratios of θ (Broken and Dotted Line) and the True Item Characteristic Function (Thick Solid Curve)

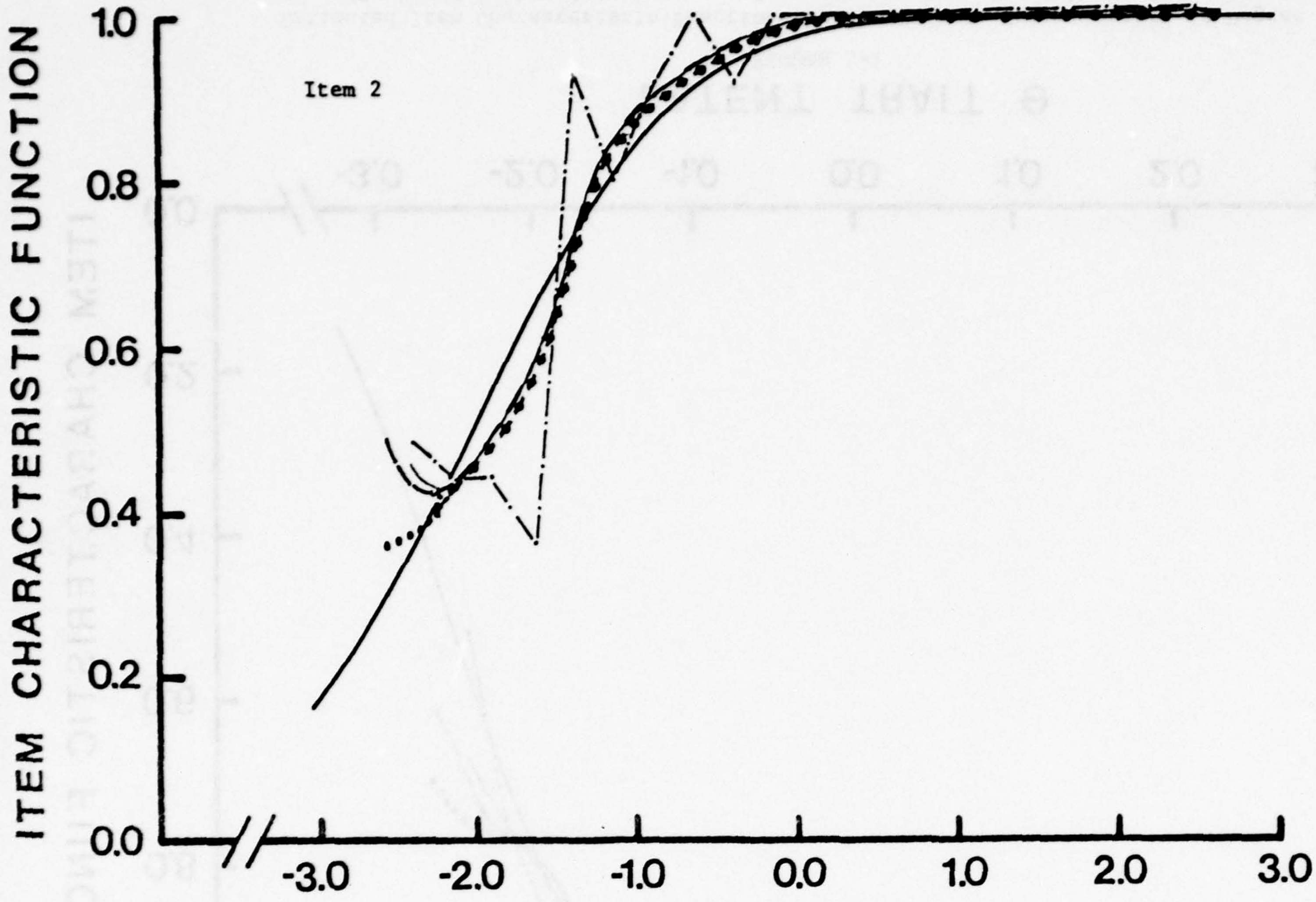
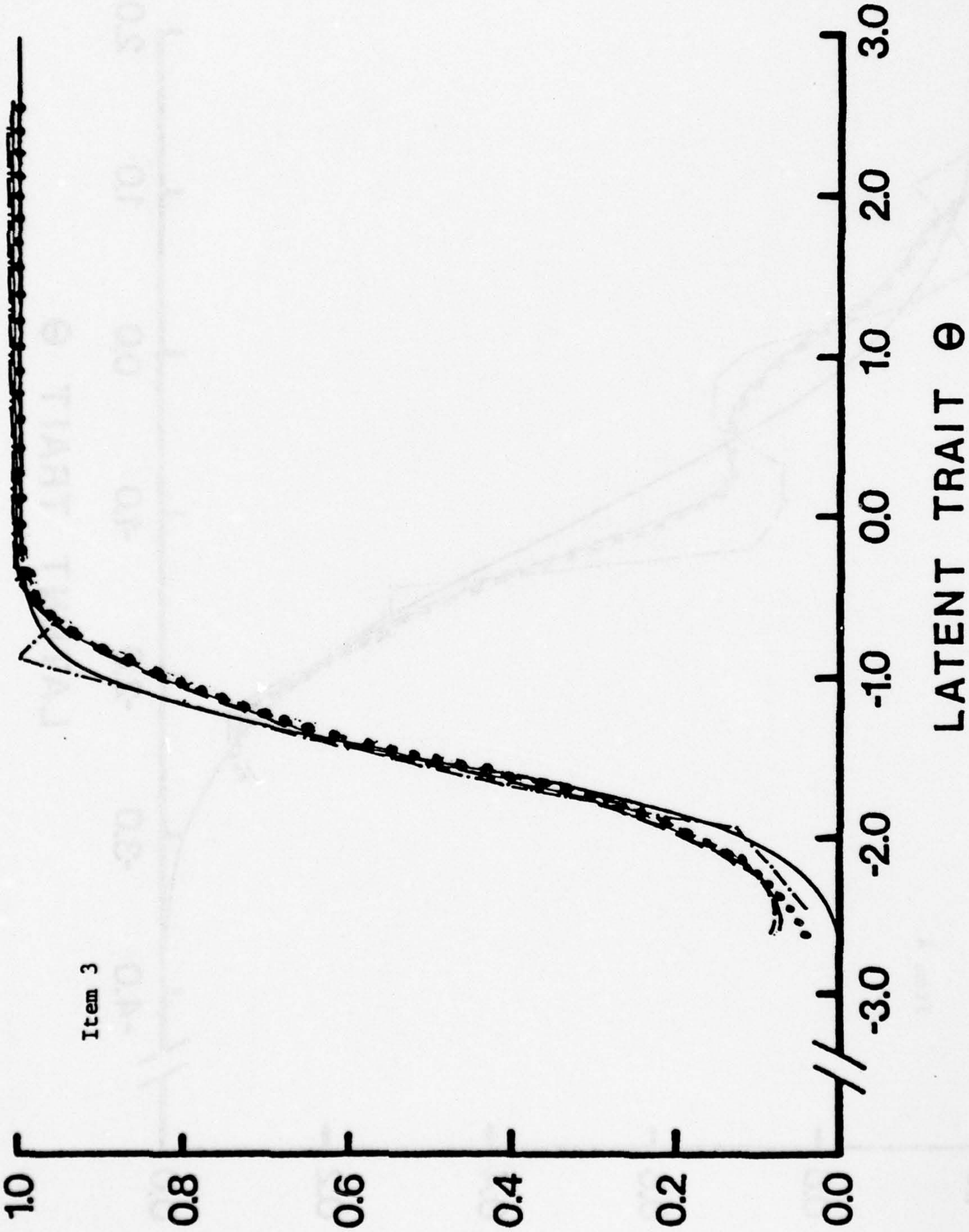


FIGURE 5-1 (Continued)



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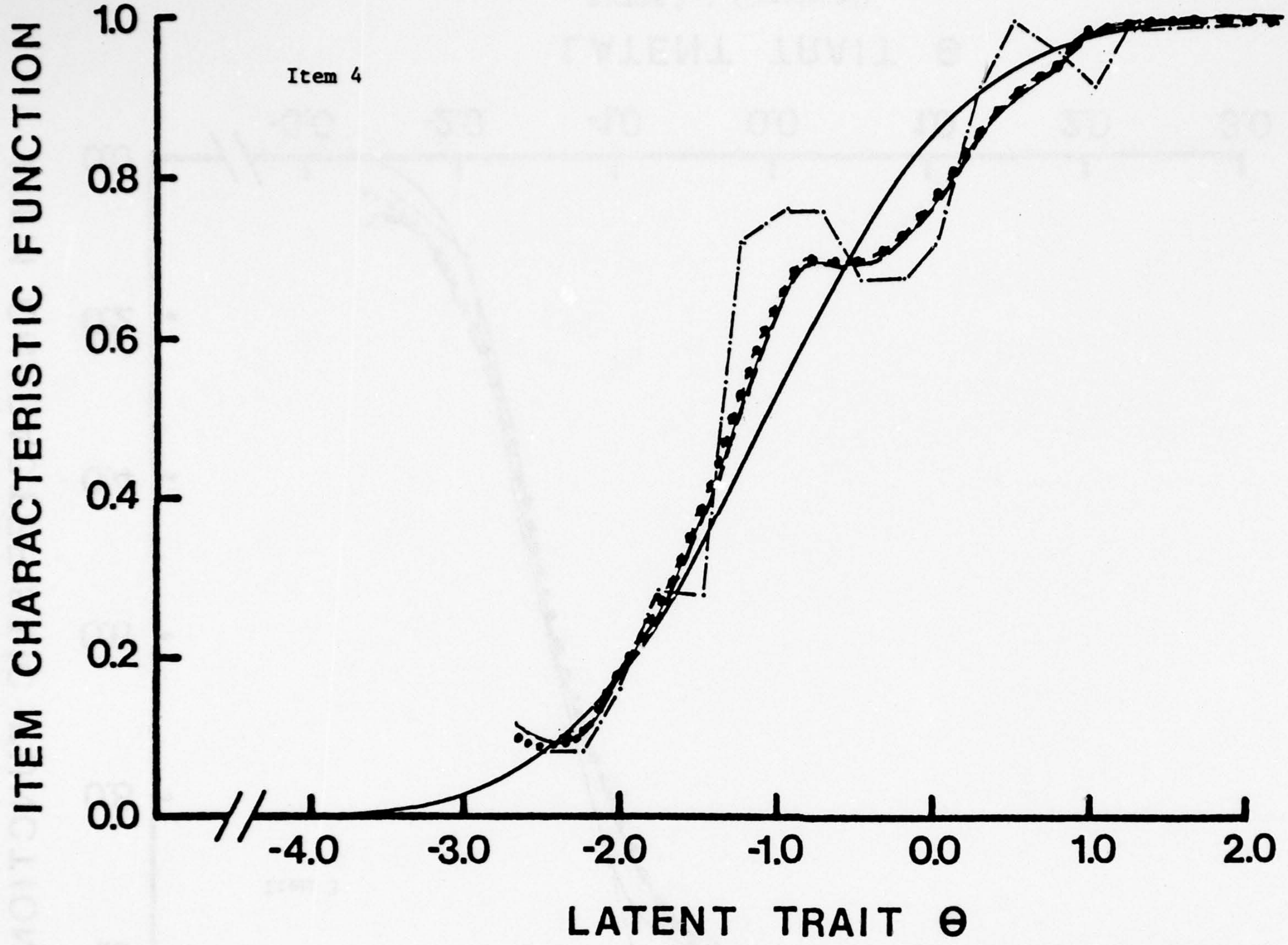
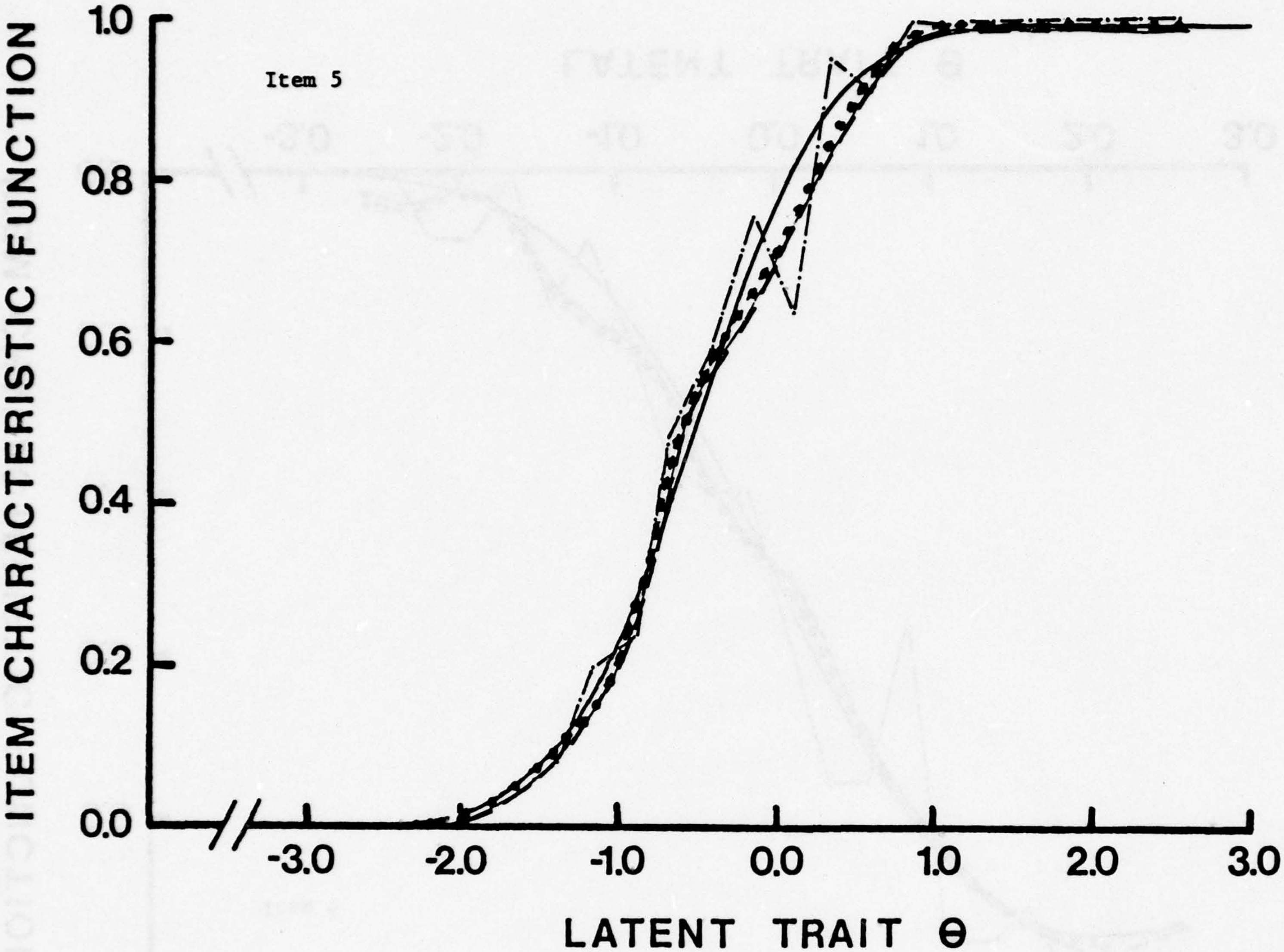


FIGURE 5-1 (Continued)



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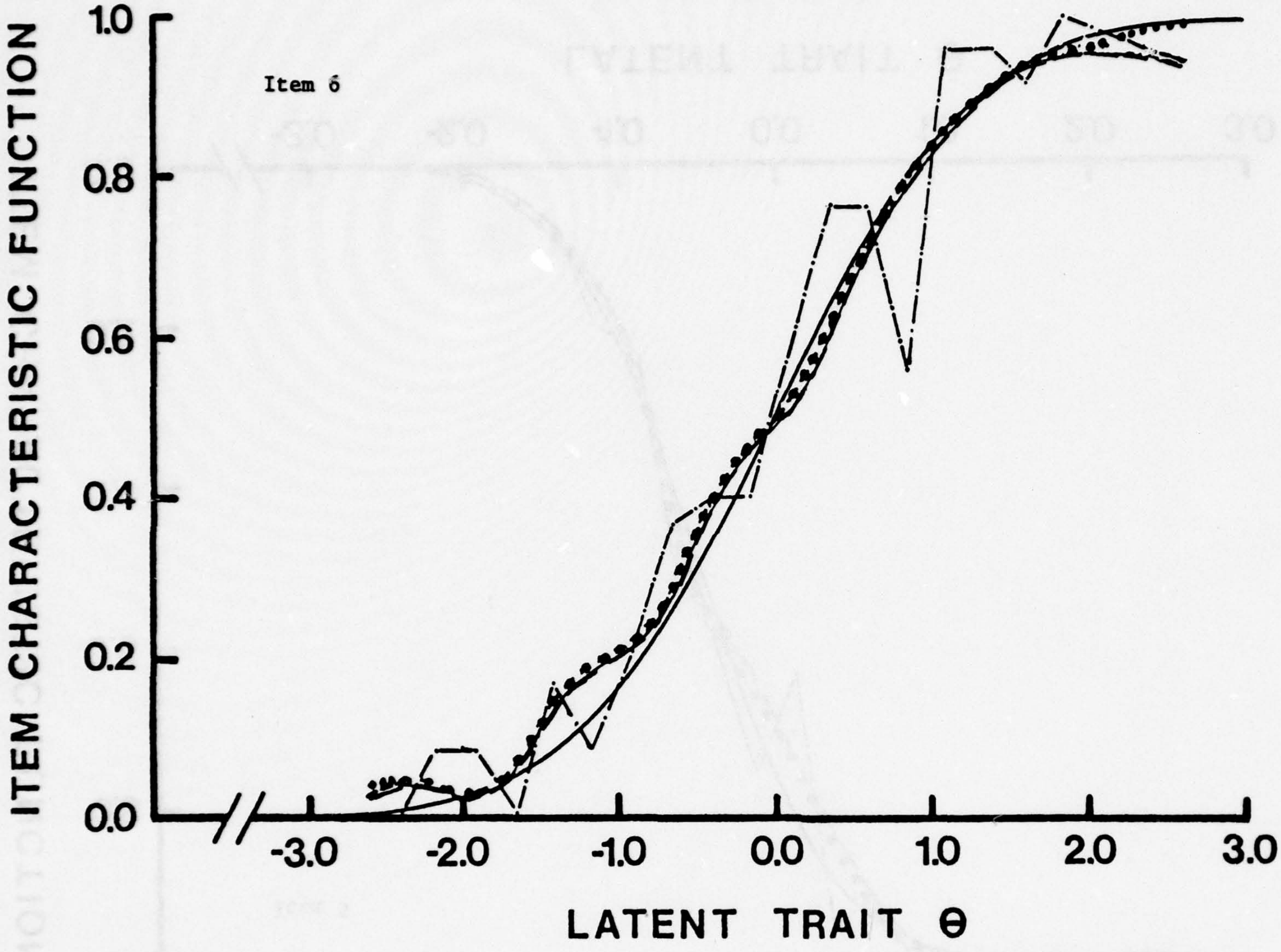

FIGURE 5-1 (Continued)

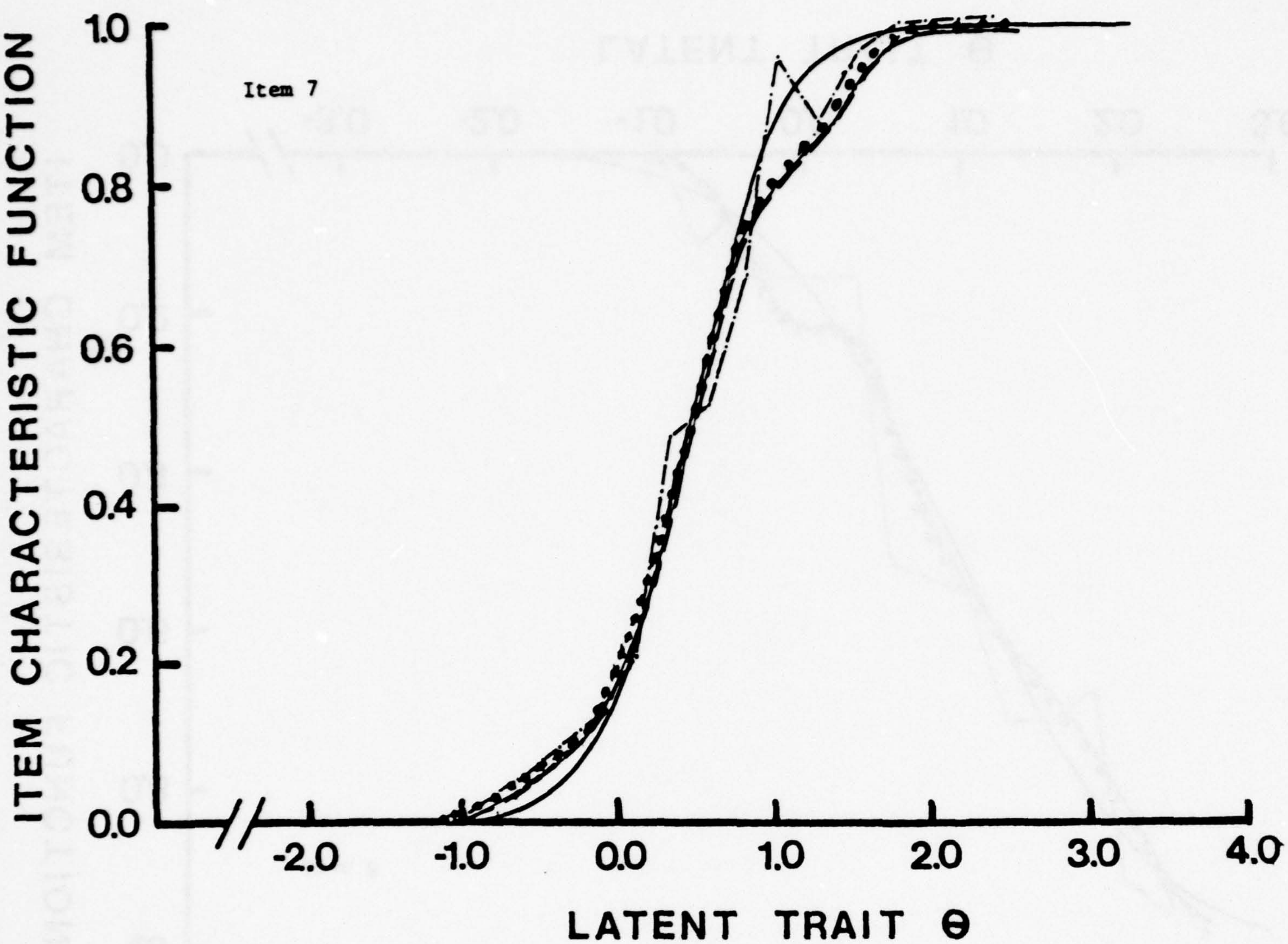

FIGURE 5-1 (Continued)

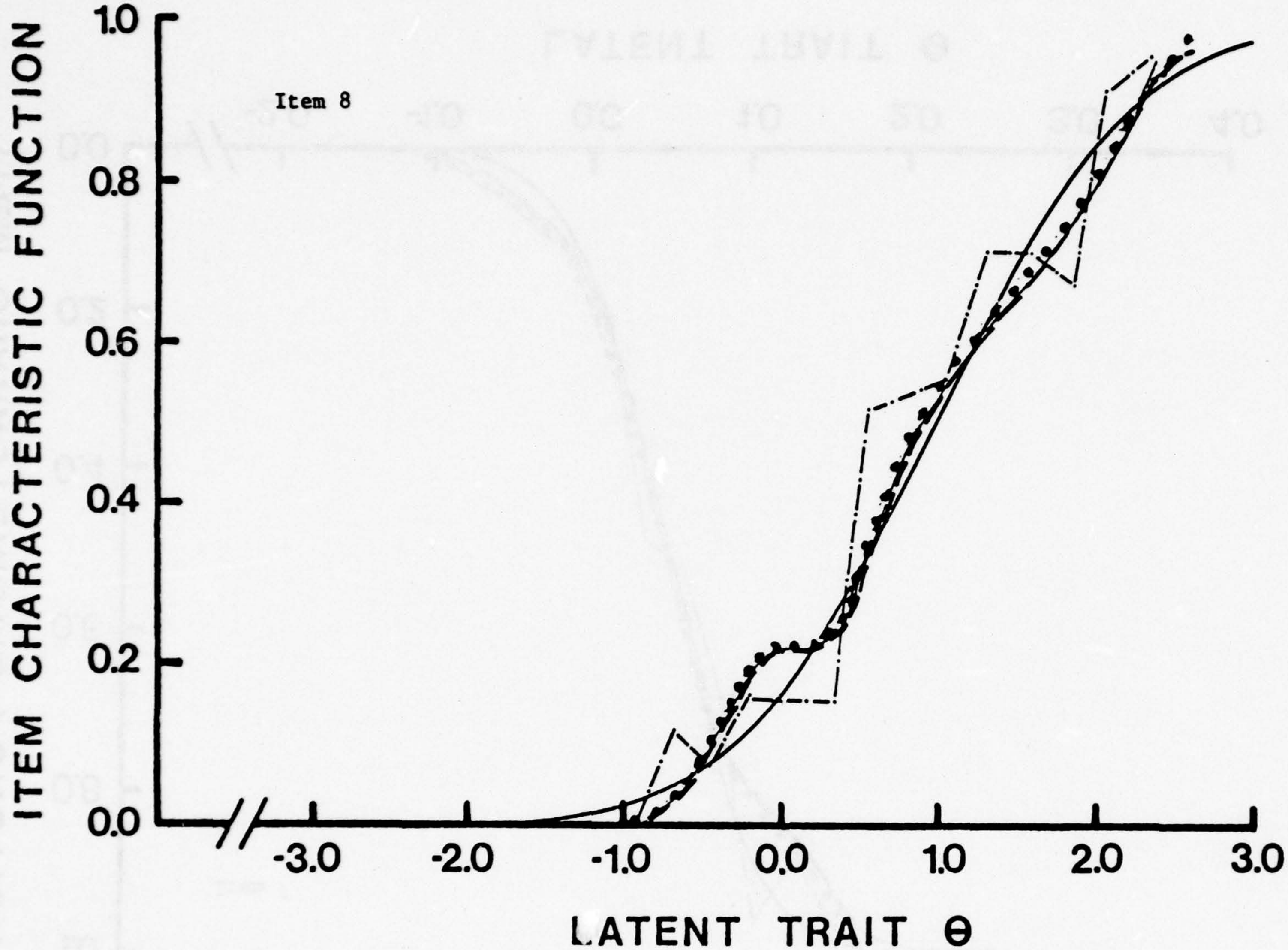

FIGURE 5-1 (Continued)

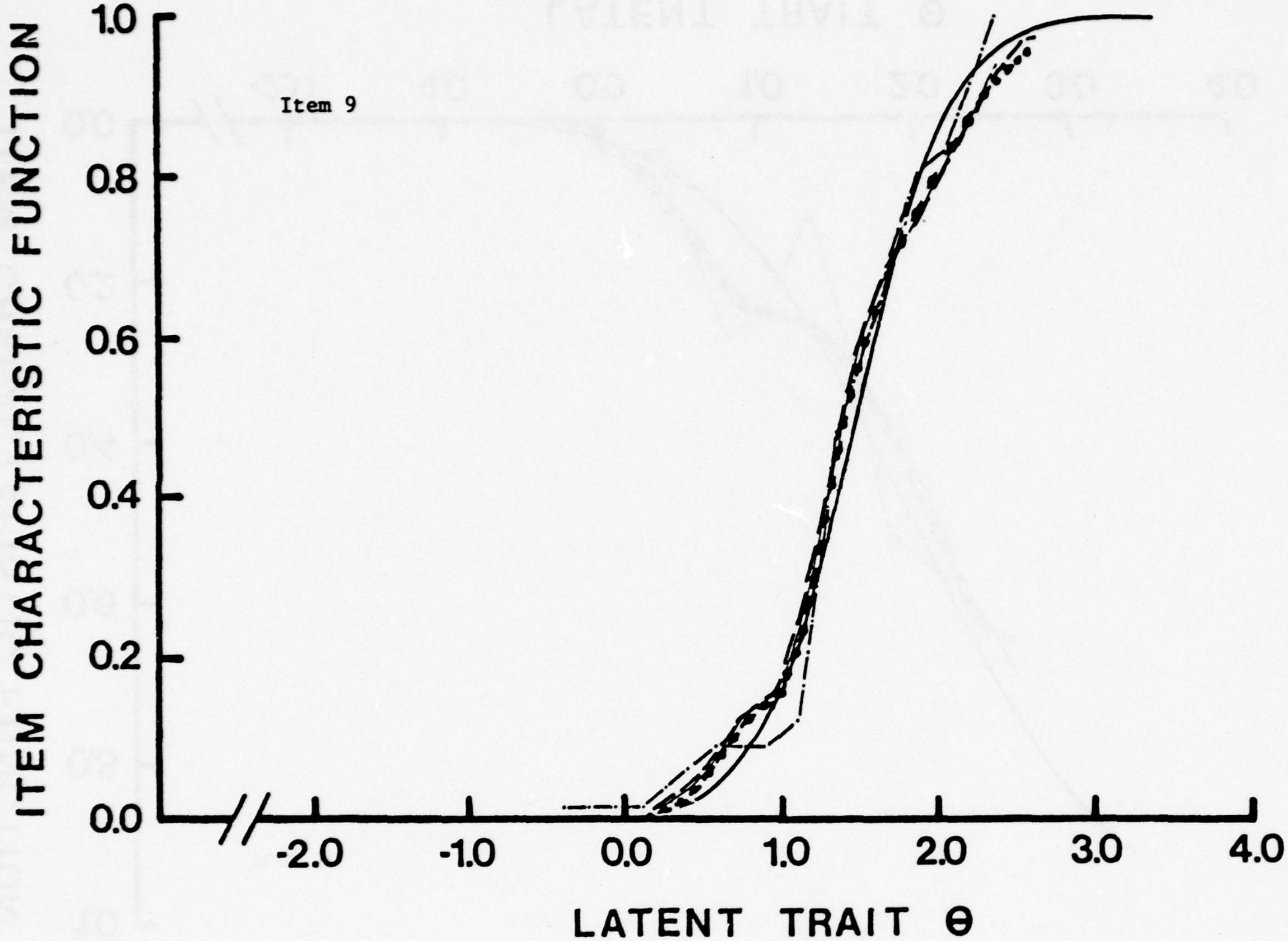

FIGURE 5-1 (Continued)

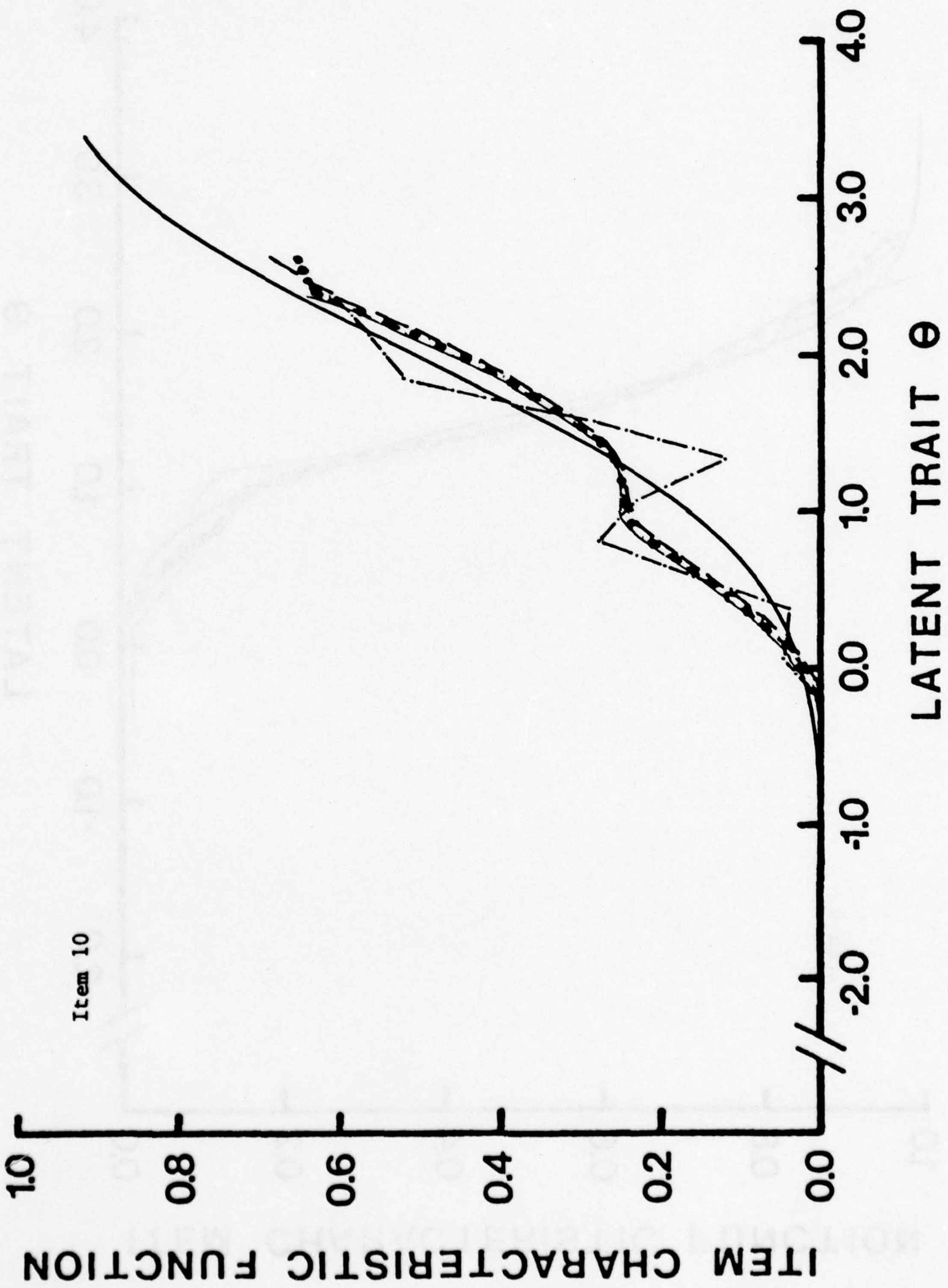

FIGURE 5-1 (Continued)

those of the Normal Approach Method not only in Degree 3 Case where only sixteen individuals have different density functions from normality, but also in Degree 4 Case where seventy nine examinees have Pearson's Type I or II density functions and three individuals are excluded. In other words, except for the range of θ outside the interval [-2.0, 2.0], these two curves are practically identical with the criterion item characteristic function, for each and every item.

Table 5-2 presents the estimated discrimination parameters for the ten binary items by the same least squares method used earlier, and the corresponding estimated difficulty parameters are shown in Table 5-3. Just as in the Normal Approach Method, these values are very close to those obtained from the criterion item characteristic functions, except for item 1. In these estimations, the interval of θ used is [-2.4, 2.4], and the range of $\hat{P}_g(\theta)$ included is [0.05, 0.95]. As usual, similar estimates were obtained by changing the range of $\hat{P}_g(\theta)$ to [0.15, 0.85], [0.10, 0.90] and [0.01, 0.99] respectively, and the results are shown in Appendix II as Tables A-2-8 and A-2-9.

These results indicate that, as far as the present set of data are concerned, the Pearson System Method works just as good as the Normal Approach Method, and the estimation has attained to the level where the Conditional P.D.F. Method can possibly attain. It is worth noting, however, that the Pearson System Method, which has more theoretical advantages than the Normal Approach Method, did not produce better results than the Normal Approach Method. In fact, there is a tendency that for very easy and very difficult items for which the

TABLE 5-2

Discrimination Parameter and Its Estimates of Each of the Ten Binary Items Obtained from the Estimated Item Characteristic Functions by the Conditional P.D.F. Method of the Pearson System Method, with the Corresponding Estimate Obtained from the Criterion Item Characteristic Function: The Range of Each Estimated Item Characteristic Function Used Is [0.05, 0.95], within the Interval of θ , [-2.4, 2.4]

METHOD / ITEM	TRUE a_g	$\hat{a}_g$ from DGR.3 0.05-0.95	$\hat{a}_g$ from DGR.4 0.05-0.95	$\hat{a}_g$ from CRITERION 0.05-0.95
1	1.5	1.177_5	1.090_5	1.400_5
2	1.0	1.053	1.097	1.024
3	2.5	1.792	1.838	1.788
4	1.0	0.883	0.880	0.868
5	1.5	1.382	1.366	1.368
6	1.0	0.905	0.897	0.895
7	2.0	1.484	1.475	1.473
8	1.0	0.887	0.905	0.886
9	2.0	1.673	1.653	1.716
10	1.0	0.714	0.707	0.725

The number of intervals used in estimation is shown as a subscript when it is less than 6 .

TABLE 5-3

Difficulty Parameter and Its Estimates of Each of the Ten Binary Items Obtained from the Estimated Item Characteristic Functions by the Conditional P.D.F. Method of the Pearson System Method, with the Corresponding Estimate Obtained from the Criterion Item Characteristic Function: The Range of Each Estimated Item Characteristic Function Used Is [0.05, 0.95], within the Interval of θ , [-2.4, 2.4]

METHOD / ITEM	TRUE b_g	$\hat{b}_g$ from DGR.3 0.05-0.95	$\hat{b}_g$ from DGR.4 0.05-0.95	$\hat{b}_g$ from CRITERION 0.05-0.95
1	-2.5	-2.788_5	-2.884_5	-2.651_5
2	-2.0	-1.966	-1.942	-2.002
3	-1.5	-1.494	-1.498	-1.507
4	-1.0	-0.990	-0.991	-1.005
5	-0.5	-0.468	-0.472	-0.472
6	0.0	-0.073	-0.073	-0.075
7	0.5	0.522	0.527	0.527
8	1.0	0.939	0.967	0.981
9	1.5	1.502	1.515	1.502
10	2.0	2.128	2.148	2.118

The number of intervals used in estimation is shown as a subscript when it is less than 6 .

difference between normal density functions and Beta density functions is expected to show, the estimation of the parameters is slightly worse in Degree 4 Case, although in Degree 3 Case these two sets of estimates are practically identical.

Figures 5-2 and 5-3 present the estimated density function of θ (dashed curve), together with the one obtained by the Two-Parameter Beta Method (broken curve), the average of the true conditional density functions (solid curve) and theoretical density function $f(\theta)$ (dashed line). Since we had to exclude three more subjects from the 486 subjects in Degree 4 Case, they are excluded from the two extreme subintervals of θ , to make the total 483.

It is observed that, in both Degree 3 and 4 Cases, the estimated density function by the Pearson System Method is slightly closer to the average of the true conditional density functions for the 500 subjects than the one obtained by the Two-Parameter Beta Method. Also the same is true if we compare them with the results obtained by the Normal Approach Method, which are shown in Figures 3-2 and 3-3.

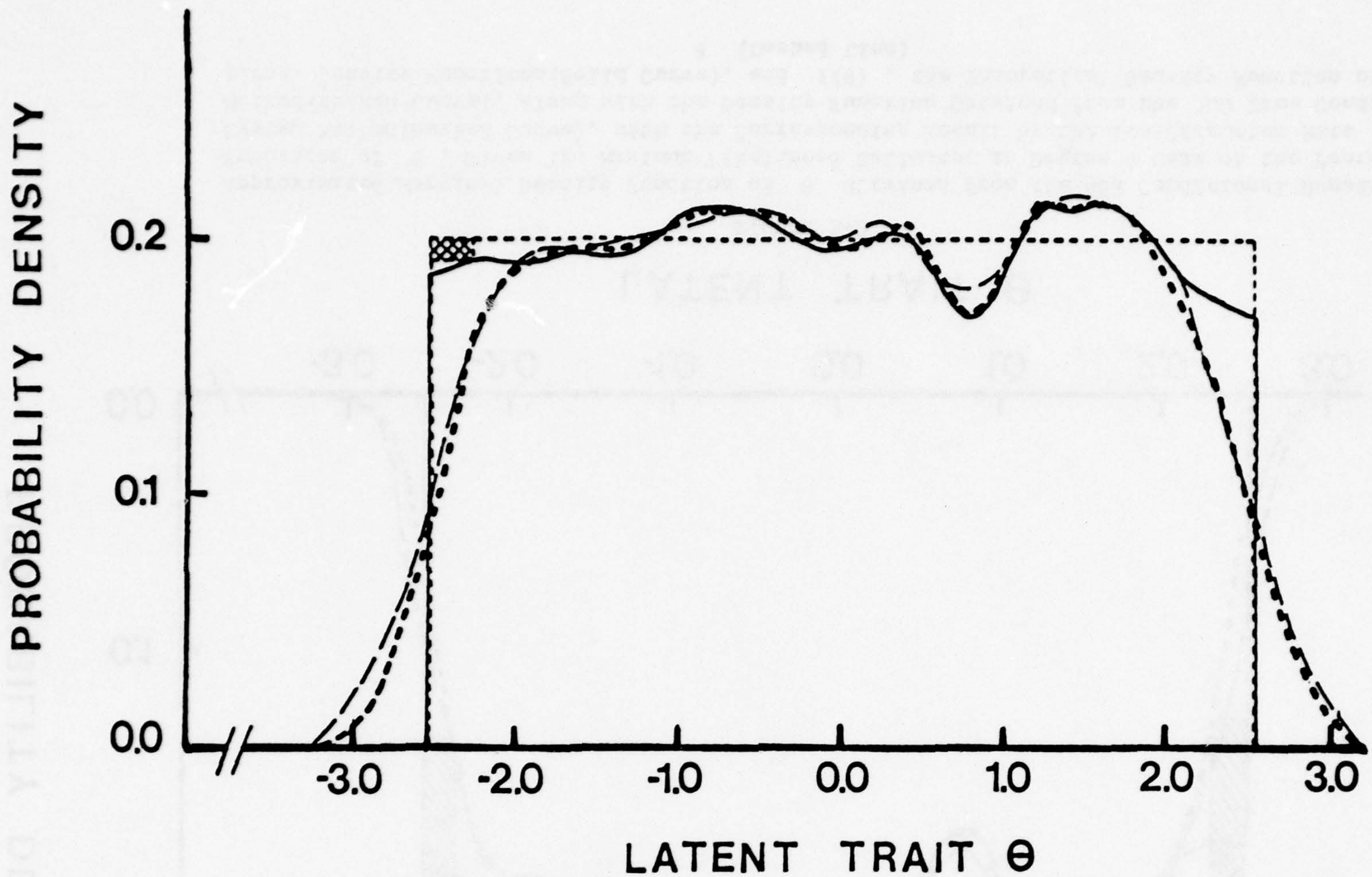

FIGURE 5-2

Approximated Marginal Density Function of θ Obtained from the 499 Conditional Density Functions of θ , Given Its Maximum Likelihood Estimate, in Degree 3 Case of the Pearson System Method (Dashed Curve), with the Corresponding Result by the Two-Parameter Beta Method (Broken Curve), Along with the Density Function Obtained from the 500 True Conditional Density Functions (Solid Curve), and $f(\theta)$, the Theoretical Density Function of θ (Dashed Line)

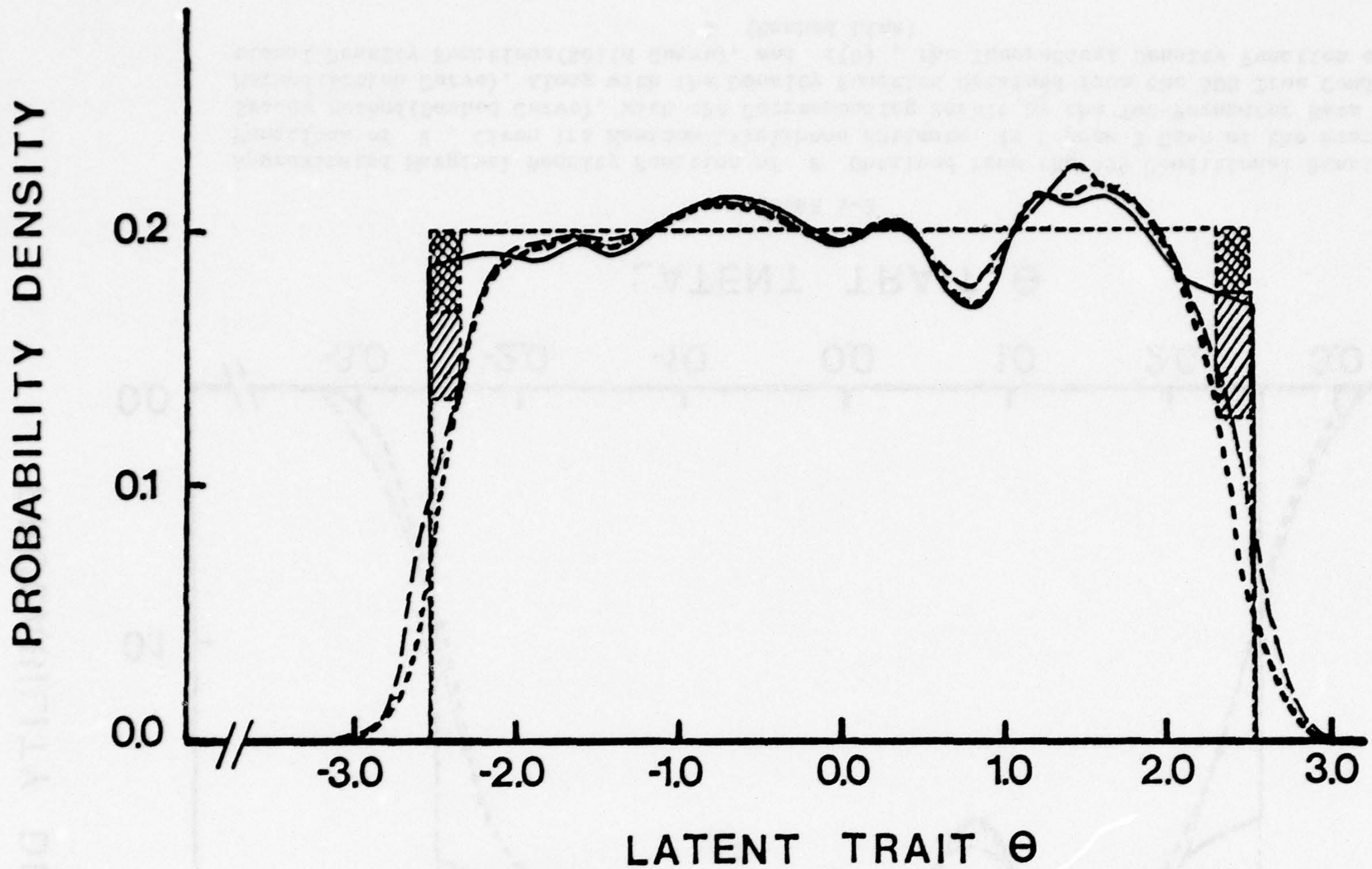

FIGURE 5-3

Approximated Marginal Density Function of θ Obtained from the 483 Conditional Density Functions of θ, Given Its Maximum Likelihood Estimate, in Degree 4 Case of the Pearson System Method (Dashed Curve), with the Corresponding Result by the Two-Parameter Beta Method (Broken Curve), Along with the Density Function Obtained from the 500 True Conditional Density Functions (Solid Curve), and $f(\theta)$, the Theoretical Density Function of θ (Dashed Line)

VI Comparison of the Approximated Conditional Density Functions

Figure 6-1 presents a typical set of the approximated conditional density functions of θ , given $\hat{\theta}$, i.e., the normal density function (dashed curve), which belongs to both the Pearson System Method and the Normal Approach Method, and the Beta density function (broken curve), which belongs to the Two-Parameter Beta Method. In this example of Subject 50, in both Degree 3 and 4 Cases, the normal density function is almost identical with the true conditional density function, which is drawn by a solid curve, while the Beta density function is flatter than them.

In contrast to the above example, Figure 6-2 presents somewhat different relationships. In this example of Subject 500, while the relationship between the normal density function and the Beta density function stays the same, they are substantially different from the true conditional density function, especially in Degree 3 Case.

In the other two examples in Figure 6-3, the normal density functions are consistently closest, including the Beta density functions which belong to the Pearson System Method (Broken and dotted curves). This is rather an interesting result, indicating that the normal approximation is better than the Beta approximation, even though the latter uses the four conditional moments rather than two.

In Appendix III, Figure A-3-1 presents 42 pairs of such comparisons, which show the general tendency. These 42 individuals were selected from the total 493 subjects used in Degree 4 Case of the Two-Parameter Beta Method, starting from Subject 1 and then picking up every twelve subject. In both graphs of Degree 3 Case and Degree 4 Case for each subject,

the true conditional density of θ , given $\hat{\theta}$, is drawn by a solid curve for the sake of comparison. From this sample of 42 subjects, it is clear that the configuration exemplified by the two graphs for Subject 50, i.e., Figure 6-1, is true for the majority of the subjects. Little discrepancy lies between the normal density curve and the true conditional density function, and the results are so similar between Degree 3 and 4 Cases, for these subjects. The configuration exemplified by Figure 6-2 for Subject 500 is recognized for subjects with extreme values of the maximum likelihood estimate in both positive and negative direction, and the estimates are much closer to the true conditional density function in Degree 4 Case than in Degree 3 Case, for these subjects. This reflects the fact that in Degree 4 Case the fit of the polynomial to $g(\hat{\theta})$ is better than in Degree 3 Case because of the use of a polynomial of a higher degree, especially at the both extreme ends of $\hat{\theta}$. This did not affect the main parts of the estimated item characteristic functions of the ten binary items, however, as we have seen in Figure 3-1 and Figure 5-1.

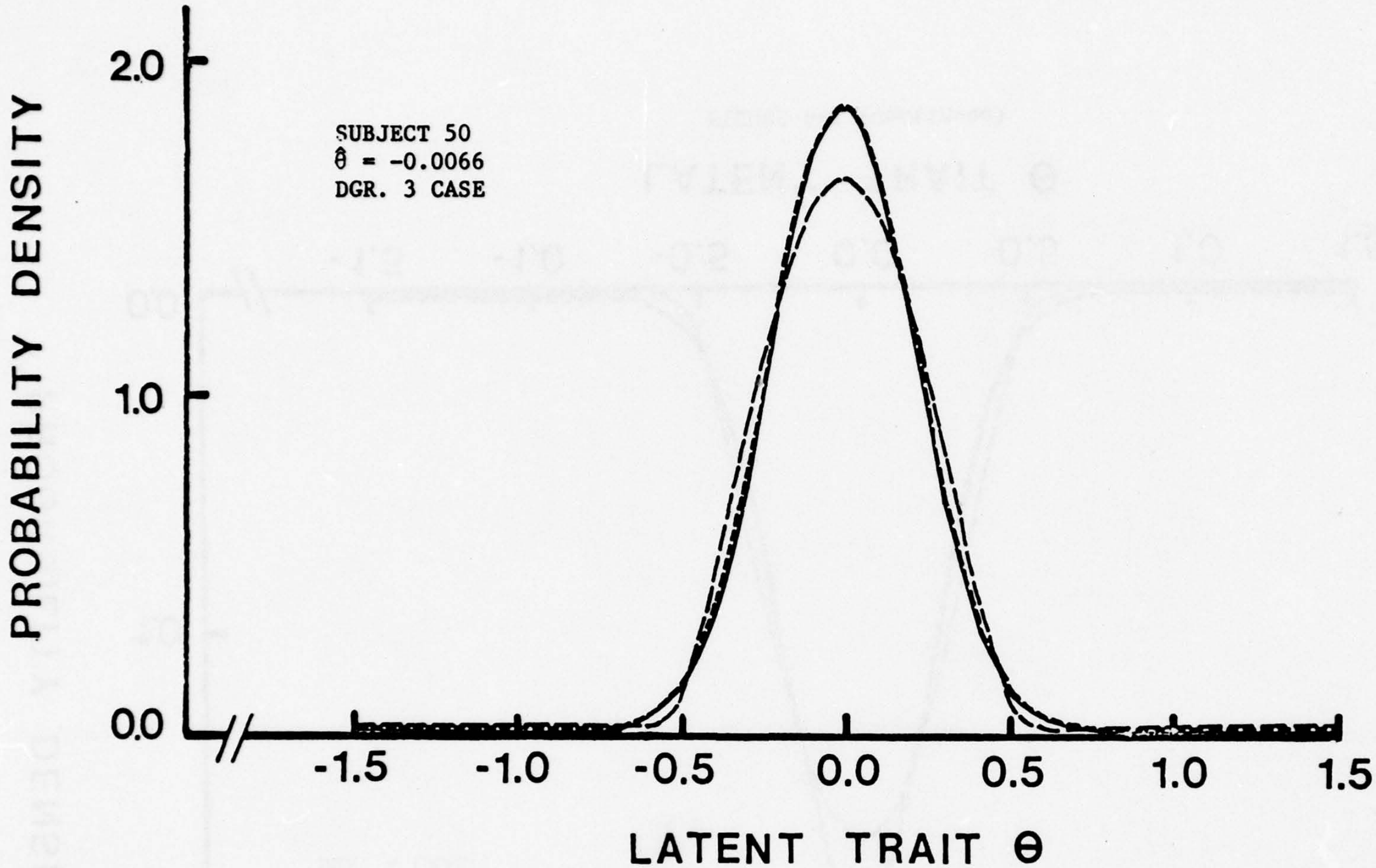

FIGURE 6-1

The conditional probability density of θ, given $\hat{\theta}$ (Solid Curve), and its estimates by the Normal Approach Method (Dashed Curve) and by the Two-Parameter Beta Method (Broken Curve).

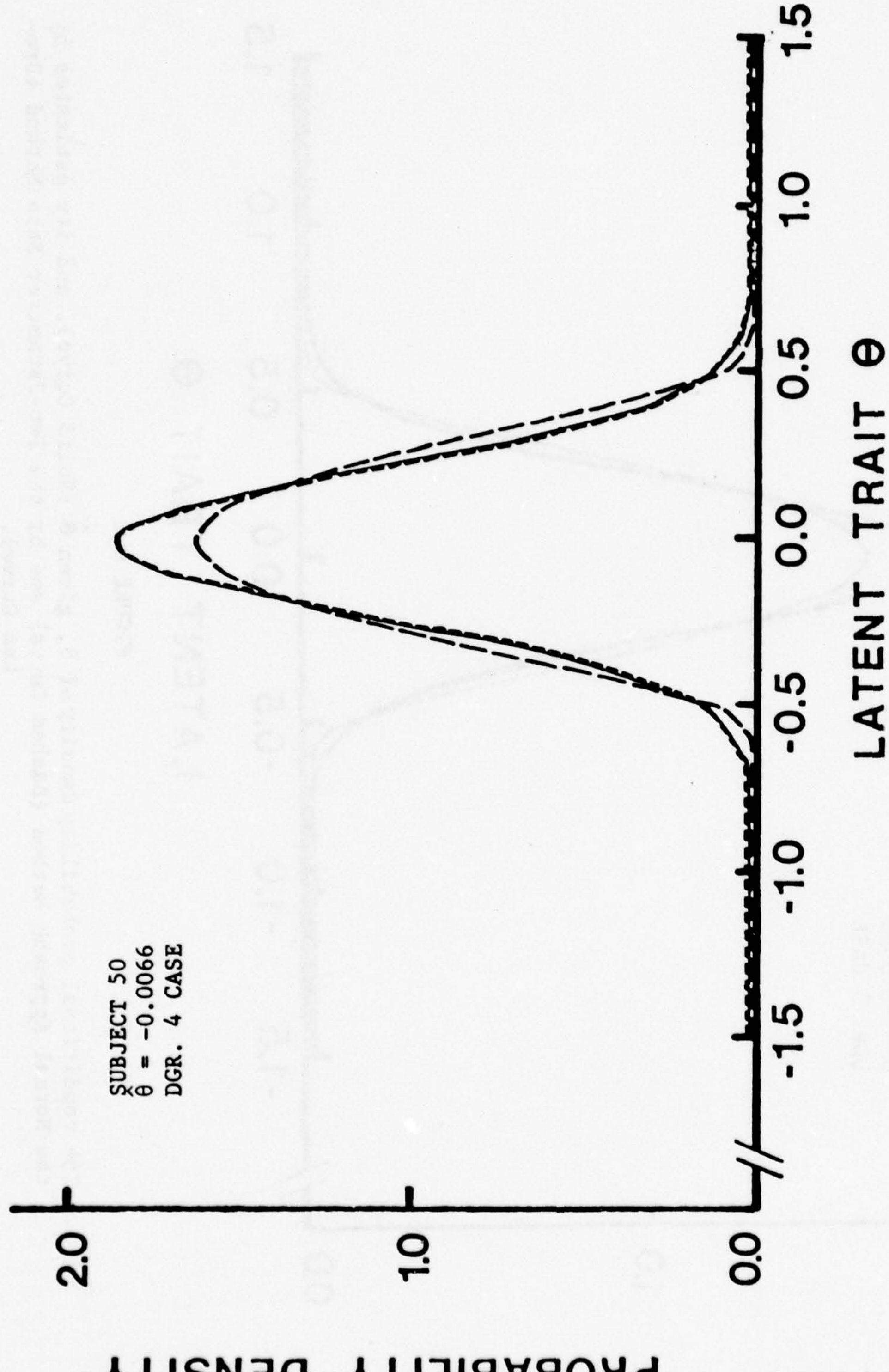

FIGURE 6-1 (Continued)

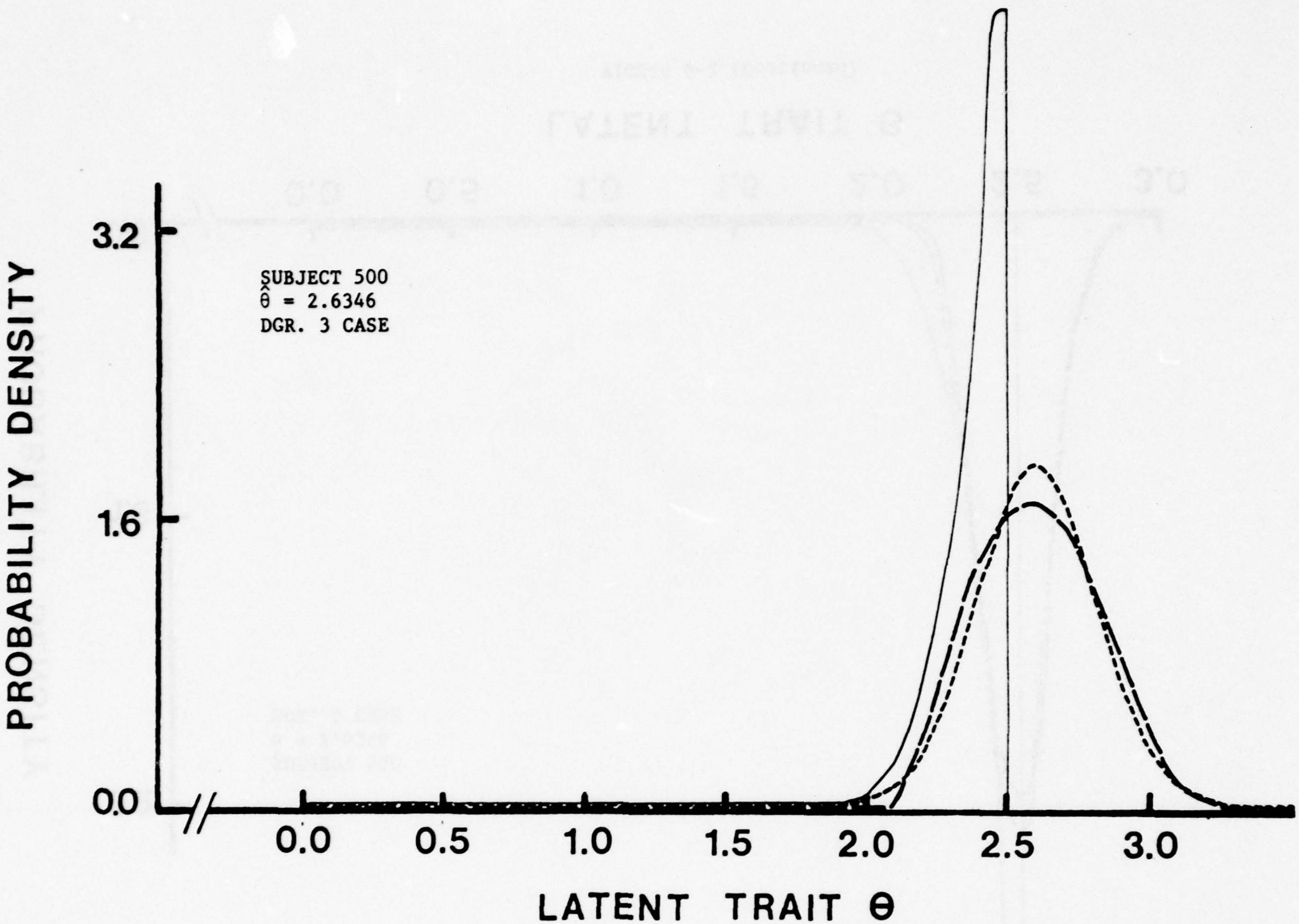

FIGURE 6-2

The conditional probability density of θ, given $\hat{\theta}$ (Solid Curve), and its estimates by the Normal Approach Method (Dashed Curve) and by the Two-Parameter Beta Method (Broken Curve).

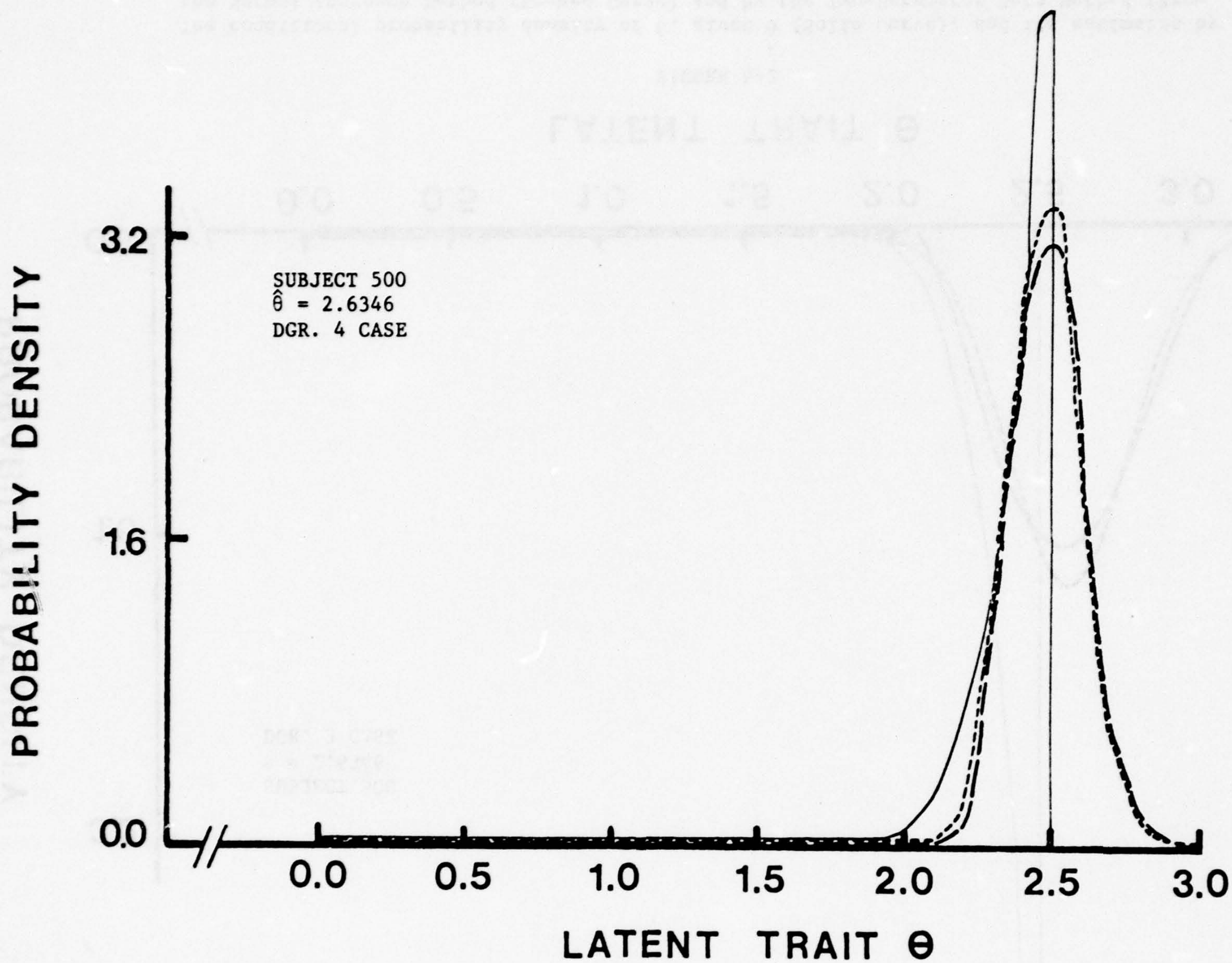

FIGURE 6-2 (Continued)

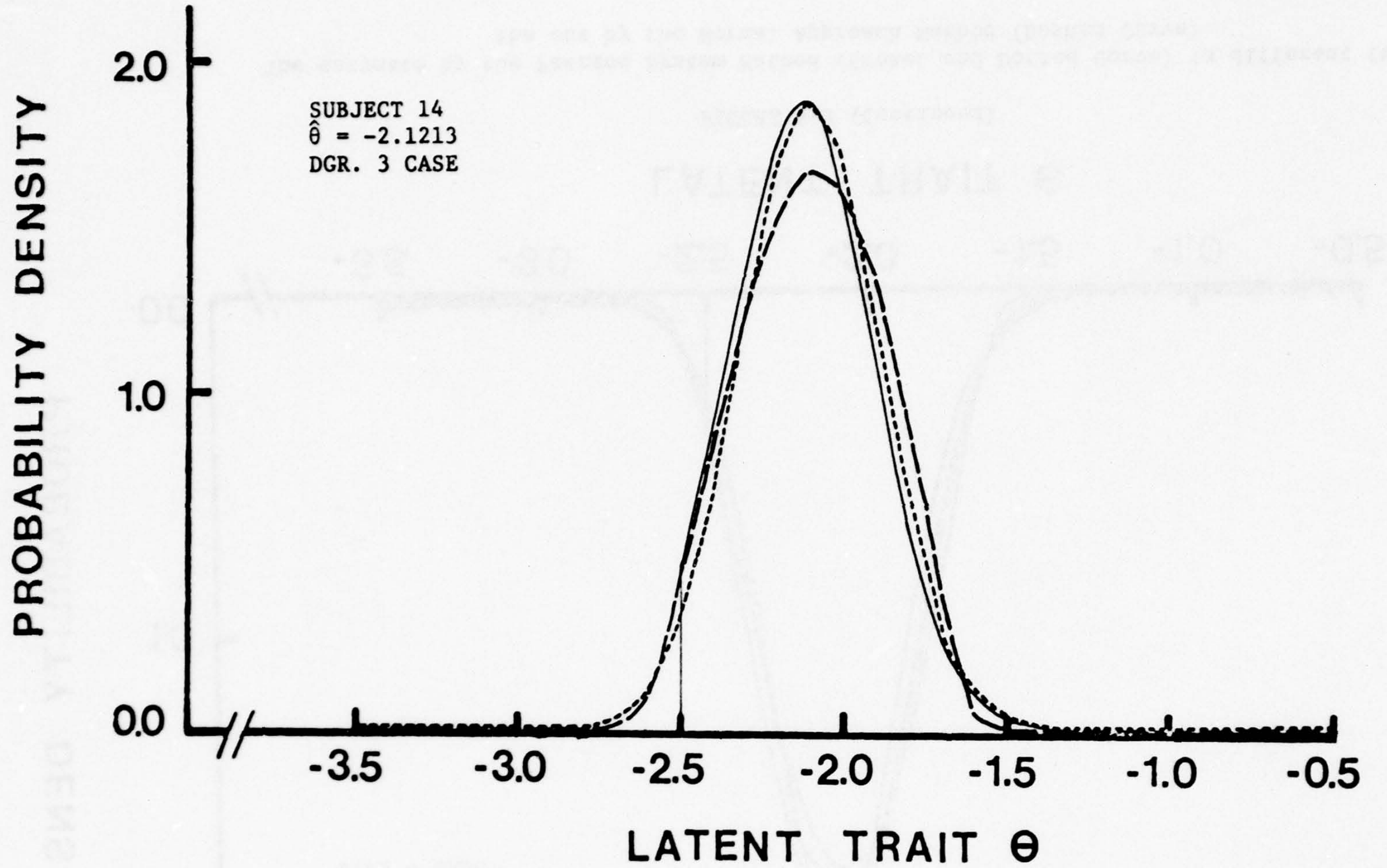

FIGURE 6-3

The conditional probability density of θ, given $\hat{\theta}$ (Solid Curve), and its estimates by the Normal Approach Method (Dashed Curve) and by the Two-Parameter Beta Method (Broken Curve). (The estimated density by the Pearson System Method is the same as the one by the Normal Approach Method.)

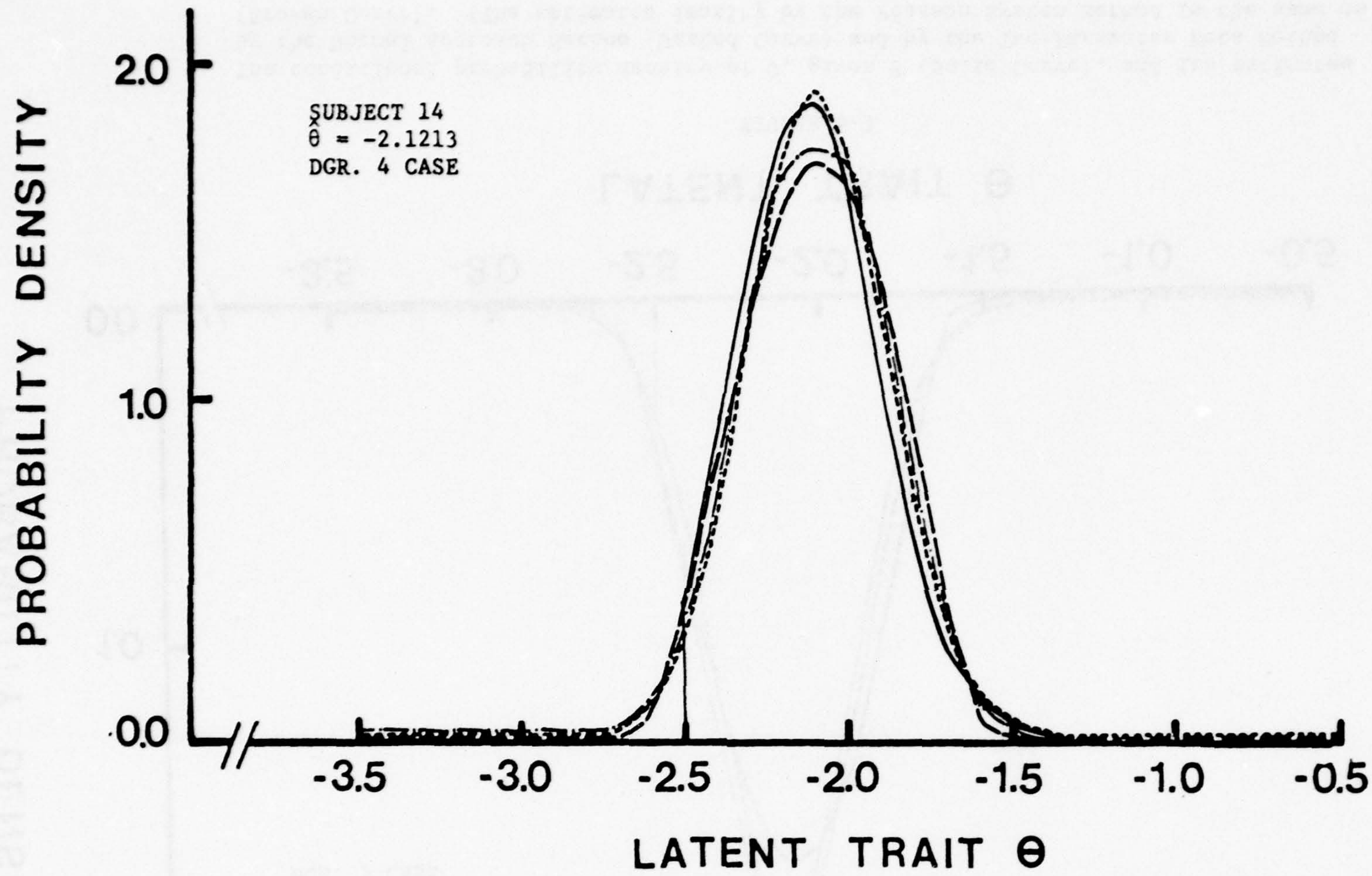

FIGURE 6-3 (Continued)

The estimate by the Pearson System Method (Broken and Dotted Curve) is different from the one by the Normal Approach Method (Lashed Curve).

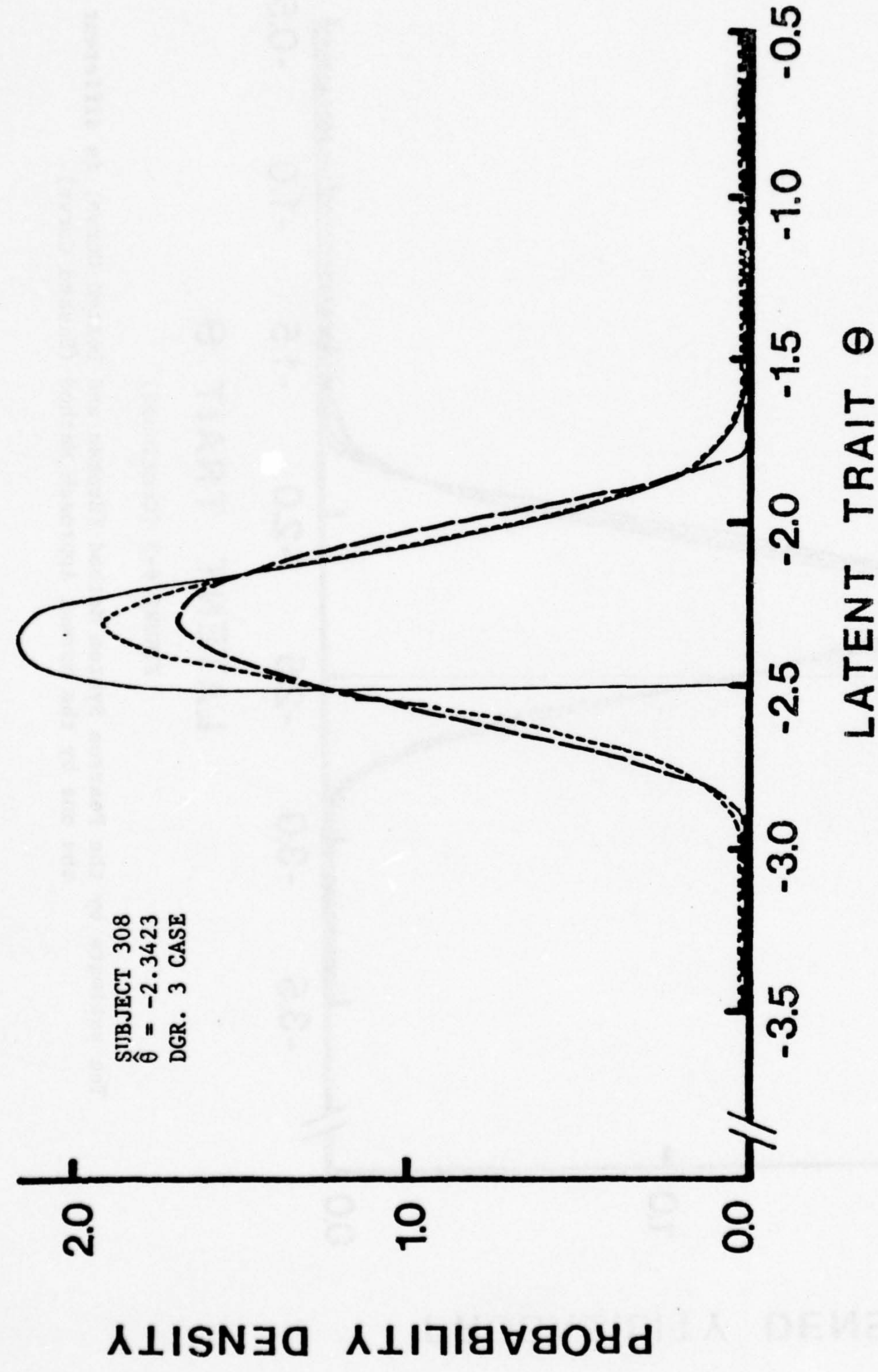

FIGURE 6-3 (Continued)

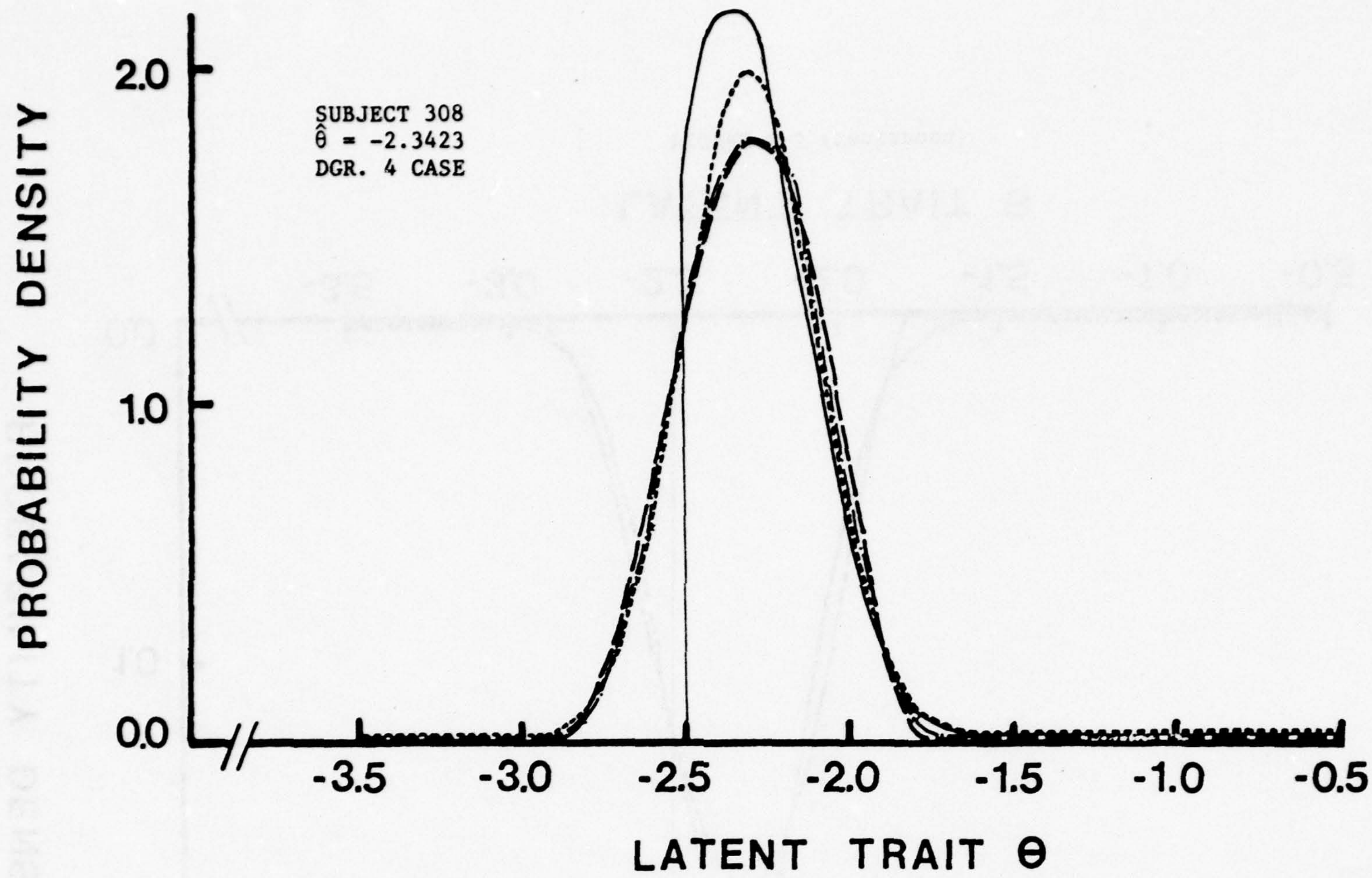

FIGURE 6-3 (Continued)

The estimate by the Pearson System Method (Broken and Dotted Curve) is different from the one by the Normal Approach Method (Dashed Curve).

VII Discussion and Conclusion

We have seen the results of the Two-Parameter Beta Method, of the Normal Approach Method, and of the Pearson System Method, all of which belong to the Conditional P.D.F. Method, in addition to those obtained by the Histogram Ratio Method (Samejima, 1977d, 1978) and by the Curve Fitting Method (Samejima, 1978) in the context of the Two-Parameter Beta Method. It turned out that all the three in the Conditional P.D.F. Method have produced highly satisfactory results as far as the present simulated data are concerned.

This is rather unexpected, considering that each method has somewhat different characteristics. It should be recalled, however, that, with the present simulated data, the majority of the conditional density functions of θ , given $\hat{\theta}$, are normal density functions in the Pearson System Method, and it is natural that both the Pearson System Method and the Normal Approach Method produce almost identical results. It is desired, therefore, to use different types of data in addition to the present ones, before we try to extract some definite conclusion about their characteristics.

The comparison of the estimated conditional density functions has revealed that, in general, the normal density function fits slightly better to the true conditional density function than the Beta density, however. This may indicate that the counterbalance of the use of the higher conditional moments and the inflexibility of the shape of the function is in favor of the latter.

Since we have succeeded in attaining the level of estimation that we can possibly hope for as far as the present data are concerned, we

should direct ourselves to overcome the limitation of the criterion operating characteristic now. One conceivable way is to try to approach the joint density function of θ and $\hat{\theta}$ for each item score group, instead of approaching the conditional density function of θ, given $\hat{\theta}$. This orientation is closer to the Normal Approximation Method which the author has tried previously (Samejima, 1977b).

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APPENDIX I

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TABLE A-1-1

Discrimination Parameter and Its Estimates of Each of the Ten Binary Items Obtained from the Estimated Item Characteristic Functions by the Conditional P.D.F. Method of the Normal Approach Method, with the Corresponding Estimate Obtained from the Criterion Item Characteristic Function, Using Each of the Four Different Ranges of the Estimated Item Characteristic Functions, [0.15, 0.85], [0.10, 0.90], [0.05, 0.95] and [0.01, 0.99], within the interval of θ , [-2.4, 2.4].

METHOD		DGR. 3				DGR. 4				CRITERION			
ITEM	TRUE a_g	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99
1	1.5	0.5676_3	0.7993_4	1.1710_5	1.5268	0.5328_3	0.7959_4	1.1795_5	1.3996	0.9536_3	1.1064_4	1.4003_5	1.5517
2	1.0	0.9030	1.0417	1.0546	1.0833	0.9565	1.0475	1.0779	1.0977	0.8595	0.9693	1.0238	1.0583
3	2.5	1.9672	1.9672	1.7931	1.9040	1.9738	1.9738	1.8460	1.8835	1.9510	1.8923	1.7876	1.8960
4	1.0	0.8327	0.8251	0.8832	0.9111	0.8251	0.8171	0.8820	0.9075	0.8181	0.8114	0.8679	0.8952
5	1.5	1.3244	1.3583	1.3823	1.4527	1.3034	1.3425	1.3657	1.4396	1.3093	1.3464	1.3682	1.4376
6	1.0	0.8703	0.8672	0.9054	0.8630	0.8623	0.8581	0.8328	0.8510	0.8415	0.8585	0.8950	0.8631
7	2.0	1.5755	1.5295	1.4844	1.4964	1.5724	1.5240	1.4755	1.4862	1.5710	1.5214	1.4725	1.4814
8	1.0	0.8291	0.8460	0.8867	0.9820	0.8288	0.8480	0.9183	0.9795	0.8248	0.8460	0.8860	0.9906
9	2.0	1.8008_5	1.7394	1.6731	1.7086	1.8411_5	1.7572	1.6853	1.7209	1.8275_5	1.7542	1.7165	1.7445
10	1.0	0.6965	0.6940	0.7142	0.8753	0.6975	0.6963	0.7169	0.8791	0.7128	0.7076	0.7255	0.8839

The number of intervals used in estimation is shown as a subscript when it is less than 6 .

TABLE A-1-2

Difficulty Parameter and Its Estimates of Each of the Ten Binary Items Obtained from the Estimated Item Characteristic Functions by the Conditional P.D.F. Method of the Normal Approach Method, with the Corresponding Estimate Obtained from the Criterion Item Characteristic Function, Using Each of the Four Different Ranges of the Estimated Item Characteristic Functions, [0.15, 0.85], [0.10, 0.90], [0.05, 0.95] and [0.01, 0.99], within the interval of θ , [-2.4, 2.4].

METHOD		DGR. 3				DGR. 4				CRITERION			
ITEM	TRUE b_g	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99
1	-2.5	-3.3951_3	-3.0680_4	-2.7925_5	-2.6330	-3.4939_3	-3.0881_4	-2.7993_5	-2.6946	-2.8510_3	-2.7703_4	-2.6506_5	-2.6001
2	-2.0	-1.9641	-1.9667	-1.9655	-1.9601	-1.9540	-1.9580	-1.9569	-1.9533	-2.0081	-2.0072	-2.0022	-1.9943
3	-1.5	-1.4762	-1.4762	-1.4938	-1.5132	-1.4853	-1.4853	-1.4970	-1.5043	-1.4896	-1.4945	-1.5068	-1.5252
4	-1.0	-1.0446	-1.0425	-0.9896	-1.0001	-1.0530	-1.0509	-0.9894	-0.9990	-1.0589	-1.0572	-1.0048	-1.0148
5	-0.5	-0.4840	-0.4696	-0.4681	-0.4785	-0.4901	-0.4741	-0.4719	-0.4795	-0.4885	-0.4732	-0.4719	-0.4817
6	0.0	-0.0460	-0.0644	-0.0727	-0.0418	-0.0441	-0.0631	-0.0483	-0.0378	-0.0531	-0.0640	-0.0747	-0.0354
7	0.5	0.4960	0.5208	0.5215	0.5133	0.5019	0.5263	0.5266	0.5175	0.5000	0.5251	0.5268	0.5191
8	1.0	0.9674	0.9614	0.9387	0.9856	0.9729	0.9631	0.9575	0.9852	0.9745	0.9645	0.9812	0.9779
9	1.5	1.4820_5	1.5014	1.5019	1.5081	1.4921_5	1.5064	1.5076	1.5136	1.4963_5	1.5088	1.5016	1.5064
10	2.0	2.1365	2.1374	2.1279	2.0472	2.1405	2.1410	2.1312	2.0497	2.1237	2.1256	2.1176	2.0408

The number of intervals used in estimation is shown as a subscript when it is less than 6 .

APPENDIX II

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TABLE A-2-1

The Estimated Four Parameters of the Beta Density Function for Each of the 6 Sbujects of Pearson Type I: Degree 3 Case

SUBJECT I.D.	ESTIMATED LATENT TRAIT	BETA1	BETA2	R	B-A	A	B	P	Q
1	-2.699	0.016	2.889	41.890	2.674	-4.241	-1.567	25.254	16.636
3	-2.672	0.012	2.865	36.368	2.497	-4.068	-1.571	21.288	15.080
4	-2.706	0.016	2.841	29.925	2.255	-3.963	-1.709	17.624	12.301
101	-2.742	0.024	2.776	20.117	1.844	-3.759	-1.914	11.919	8.198
201	-2.742	0.024	2.776	20.117	1.844	-3.759	-1.914	11.919	8.198
401	-2.706	0.016	2.841	29.925	2.255	-3.963	-1.708	17.624	12.301

TABLE A-2-2

The Estimated Four Parameters of the Beta Density Function for Each of the 10 Subjects of Pearson Type II: Degree 3 Case

SUBJECT I.D.	ESTIMATED LATENT TRAIT	BETA1	BETA2	R	B-A	A	B	P	Q
102	-2.507	0.002	2.976	215.256	6.156	-6.045	0.110	125.462	89.794
104	-2.569	0.003	2.948	101.953	4.205	-4.925	-0.721	58.478	43.475
105	-2.467	0.002	2.975	208.087	6.076	-5.947	0.129	120.776	87.311
109	-2.476	0.002	2.988	408.872	8.603	-7.687	0.915	249.957	158.915
202	-2.473	0.002	2.988	408.872	8.603	-7.684	0.918	249.957	158.915
204	-2.450	0.001	2.961	143.767	5.021	-5.169	-0.147	79.146	64.621
206	-2.419	0.001	2.987	401.980	8.471	-7.311	1.160	234.196	167.784
209	2.872	0.001	3.001	6074.457	38.665	-5.961	32.704	1379.876	4694.578
300	2.856	0.001	2.987	401.980	8.471	-0.728	7.743	167.784	234.196
302	-2.488	0.002	2.948	105.738	4.304	-4.843	-0.539	59.053	46.685

TABLE A-2-3

The Estimated Four Parameters of the Beta Density Function for Each of the 41 Sbujects of Pearson Type I: Degree 4 Case

SUBJECT I.D.	ESTIMATED LATENT TRAIT	BETA1	BETA2	R	B-A	A	B	P	Q
5	-2.363	0.016	2.655	32.686	2.354	-3.707	-1.353	19.376	13.310
94	2.272	0.010	2.912	55.757	3.089	0.966	4.055	22.743	33.014
96	2.357	0.029	2.752	17.676	1.730	1.600	3.330	7.145	10.530
97	2.253	0.009	2.929	67.897	3.426	0.838	4.264	27.175	40.722
100	2.518	0.494	1.476	-0.024	0.367	2.178	2.545	-0.016	-0.008
102	-2.507	0.138	2.325	4.042	0.862	-2.965	-2.102	2.511	1.531
104	-2.569	0.531	1.495	-0.047	0.364	-2.592	-2.228	-0.015	-0.031
105	-2.467	0.074	2.613	9.279	1.261	-3.175	-1.914	5.716	3.562
108	-2.321	0.310	2.912	55.757	3.089	-4.104	-1.015	33.014	22.743
109	-2.476	0.084	2.518	7.072	1.110	-3.085	-1.975	4.335	2.738
192	2.502	0.332	1.855	0.955	0.523	2.226	2.749	0.338	0.617
194	2.456	0.137	2.395	4.657	0.917	2.035	2.951	1.745	2.912
198	2.515	0.499	1.485	-0.018	0.368	2.177	2.546	-0.012	-0.006
200	2.223	0.007	2.917	61.532	3.262	0.820	4.082	25.701	35.831
202	-2.473	0.075	2.580	8.471	1.204	-3.141	-1.937	5.199	3.272
204	-2.450	0.056	2.674	11.822	1.417	-3.251	-1.834	7.228	4.593
206	-2.419	0.035	2.727	15.596	1.627	-3.333	-1.707	9.339	6.256
208	-2.257	0.005	2.947	96.335	4.086	-4.616	-0.529	56.526	39.809
209	-2.323	0.010	2.865	37.356	2.523	-3.733	-1.210	21.556	15.800
210	-2.311	0.009	2.884	43.040	2.717	-3.846	-1.129	25.017	18.023
288	2.238	0.007	2.901	51.730	2.980	0.935	3.915	21.893	29.837
293	2.360	0.029	2.752	17.676	1.730	1.602	3.332	7.145	10.530
294	2.358	0.056	2.659	11.343	1.391	1.790	3.180	4.432	6.912
298	2.219	0.007	2.917	61.532	3.262	0.816	4.079	25.701	35.831
302	-2.488	0.097	2.511	6.684	1.078	-3.081	-2.002	4.134	2.550
303	-2.313	0.009	2.858	48.604	2.885	-3.959	-1.074	28.491	20.113
305	-2.352	0.016	2.875	37.579	2.533	-3.816	-1.283	22.459	15.120
308	-2.342	0.012	2.846	31.944	2.333	-3.650	-1.317	18.574	13.370
392	2.343	0.024	2.827	25.837	2.090	1.457	3.547	10.272	15.565
395	2.384	0.042	2.729	15.206	1.602	1.691	3.293	5.983	9.223
396	2.264	0.009	2.898	48.604	2.885	1.026	3.911	20.113	28.491
397	2.513	0.451	1.644	0.285	0.423	2.276	2.698	0.097	0.188
398	2.453	0.134	2.367	4.434	0.900	2.037	2.936	1.674	2.761
400	2.256	0.009	2.884	43.040	2.717	1.075	3.792	18.023	25.017
404	-2.371	0.020	2.820	25.795	2.094	-3.559	-1.465	15.282	10.514
406	-2.352	0.016	2.875	37.579	2.533	-3.816	-1.283	22.459	15.120
409	-2.315	0.009	2.898	48.604	2.885	-3.961	-1.076	28.491	20.113
494	2.344	0.024	2.827	25.837	2.090	1.457	3.547	10.272	15.565
495	2.287	0.012	2.894	45.290	2.782	1.112	3.894	18.347	26.943
498	2.436	0.096	2.497	6.491	1.065	1.955	3.020	2.487	4.004
499	2.427	0.084	2.518	7.072	1.110	1.926	3.036	2.738	4.335

TABLE A-2-4

The Estimated Four Parameters of the Beta Density Function for Each of the 41 Subjects of Pearson Type II: Degree 4 Case

SUBJECT I.D.	ESTIMATED LATENT TRAIT	BETA1	BETA2	R	B-A	A	B	P	Q
6	-2.245	0.005	2.933	77.592	3.666	-4.326	-0.661	44.856	32.736
7	-2.205	0.003	2.976	205.031	6.023	-5.796	0.228	123.389	81.642
10	-2.222	0.003	2.948	101.953	4.205	-4.598	-0.393	58.478	43.475
11	-2.098	0.001	2.947	106.976	4.336	-4.429	-0.093	58.156	48.820
12	-2.221	0.003	2.948	101.953	4.205	-4.597	-0.392	58.478	43.475
13	-2.152	0.002	2.962	141.989	4.988	-4.953	0.034	80.594	61.395
14	-2.121	0.002	2.975	208.087	6.076	-5.620	0.456	120.776	87.311
92	2.105	0.002	2.962	141.989	4.988	-0.082	4.906	61.395	80.594
93	2.051	0.001	2.947	106.976	4.336	0.046	4.382	48.820	58.156
95	2.114	0.002	2.976	215.256	6.156	-0.485	5.671	89.794	125.462
103	-2.085	0.001	2.987	401.980	8.471	-6.995	1.476	234.196	167.784
106	-2.078	0.001	2.987	401.980	8.471	-6.989	1.482	234.196	167.784
107	-2.121	0.002	2.975	208.087	6.076	-5.620	0.456	120.776	87.311
110	-2.240	0.005	2.919	64.877	3.353	-4.120	-0.766	37.081	27.795
113	-2.184	0.003	2.948	102.709	4.239	-4.580	-0.341	58.834	43.875
118	-2.207	0.003	2.920	67.630	3.429	-4.093	-0.665	37.884	29.747
186	2.140	0.003	2.948	102.709	4.239	0.296	4.535	43.875	58.834
187	2.045	0.001	2.947	106.976	4.336	0.041	4.377	48.820	58.156
193	2.021	0.001	2.973	207.162	6.055	-0.669	5.386	91.214	115.948
195	2.027	0.001	2.987	401.980	8.471	-1.533	6.938	167.784	234.196
197	2.102	0.002	2.962	141.989	4.988	-0.084	4.904	61.395	80.594
203	-2.225	0.003	2.948	101.953	4.205	-4.601	-0.396	58.478	43.475
205	-2.074	0.001	2.987	401.980	8.471	-6.985	1.486	234.196	167.784
292	2.090	0.002	2.934	84.112	3.843	0.340	4.183	37.662	46.450
295	2.049	0.001	2.947	106.976	4.336	0.045	4.381	48.820	58.156
301	-2.235	0.005	2.905	55.684	3.110	-3.960	-0.850	31.540	24.144
306	-2.214	0.003	2.934	81.378	3.757	-4.305	-0.548	46.050	35.327
313	-2.074	0.001	2.987	401.980	8.471	-6.985	1.486	234.196	167.784
391	2.102	0.002	2.962	141.989	4.988	-0.084	4.904	61.395	80.594
393	2.173	0.003	2.948	101.953	4.205	0.344	4.549	43.475	58.478
394	2.050	0.001	2.947	106.976	4.336	0.045	4.381	48.820	58.156
399	2.196	0.005	2.933	77.592	3.666	0.612	4.277	32.736	44.856
403	-2.224	0.003	2.948	101.953	4.205	-4.599	-0.395	58.478	43.475
405	-2.235	0.005	2.905	55.684	3.110	-3.960	-0.850	31.540	24.144
407	-2.220	0.003	2.934	81.378	3.757	-4.311	-0.553	46.050	35.327
408	-2.074	0.001	2.987	401.980	8.471	-6.985	1.486	234.196	167.784
491	2.176	0.003	2.948	101.953	4.205	0.347	4.552	43.475	58.478
492	2.194	0.005	2.919	64.877	3.353	0.720	4.073	27.795	37.081
493	2.195	0.005	2.933	77.592	3.666	0.611	4.277	32.736	44.856
496	2.134	0.003	2.948	102.709	4.239	0.291	4.530	43.875	58.834
497	2.169	0.003	2.934	81.378	3.757	0.503	4.260	35.327	46.050

TABLE A-2-5

Comparison of Pearson's Type I Density Function with the One Approximated by Stirling's Formula When the Parameter Values Are Relatively Small: 6 Subjects, Degree 3 Case.

<u>Subject</u> 1

θ	True P.D.F.	Approximation
-4.000	0.00000	0.00000
-3.800	0.00000	0.00000
-3.600	0.00001	0.00001
-3.400	0.00140	0.00141
-3.200	0.04065	0.04091
-3.000	0.37392	0.37627
-2.800	1.33535	1.34373
-2.600	1.96028	1.97258
-2.400	1.10086	1.10776
-2.200	0.18300	0.18415
-2.000	0.00464	0.00467
-1.800	0.00000	0.00000
-1.600	0.00000	0.00000

<u>Subject</u> 3

θ	True P.D.F.	Approximation
-4.000	0.00000	0.00000
-3.800	0.00000	0.00000
-3.600	0.00000	0.00000
-3.400	0.00073	0.00074
-3.200	0.02926	0.02947
-3.000	0.31107	0.31327
-2.800	1.21297	1.22157
-2.600	1.94428	1.95806
-2.400	1.23935	1.24814
-2.200	0.25327	0.25506
-2.000	0.00915	0.00921
-1.800	0.00001	0.00001
-1.600	0.00000	0.00000

TABLE A-2-5 (Continued)

Subject 4

θ	True P.D.F.	Approximation
-3.800	0.00000	0.00000
-3.600	0.00000	0.00000
-3.400	0.00113	0.00114
-3.200	0.04240	0.04277
-3.000	0.39882	0.40232
-2.800	1.37067	1.38268
-2.600	1.94473	1.96177
-2.400	1.07214	1.08154
-2.200	0.16738	0.16885
-2.000	0.00272	0.00275
-1.800	0.00000	0.00000

Subject 101

θ	True P.D.F.	Approximation
-3.600	0.00000	0.00000
-3.400	0.00137	0.00139
-3.200	0.06110	0.06190
-3.000	0.51138	0.51808
-2.800	1.52044	1.54037
-2.600	1.90820	1.93320
-2.400	0.90790	0.91979
-2.200	0.08937	0.09054
-2.000	0.00006	0.00006

TABLE A-2-5 (Continued)

__Subject__ 201

θ	True P.D.F.	Approximation
-3.600	0.00000	0.00000
-3.400	0.00137	0.00139
-3.200	0.06110	0.06190
-3.000	0.51138	0.51808
-2.800	1.52044	1.54037
-2.600	1.90820	1.93320
-2.400	0.90790	0.91979
-2.200	0.08937	0.09054
-2.000	0.00006	0.00006

__Subject__ 401

θ	True P.D.F.	Approximation
-3.800	0.00000	0.00000
-3.600	0.00000	0.00000
-3.400	0.00112	0.00113
-3.200	0.04222	0.04259
-3.000	0.39780	0.40129
-2.800	1.36905	1.38105
-2.600	1.94500	1.96205
-2.400	1.07397	1.08339
-2.200	0.16806	0.16954
-2.000	0.00275	0.00277
-1.800	0.00000	0.00000

TABLE A-2-6

Comparison of Pearson's Type I Density Function with the One Approximated by Stirling's Formula When the Parameter Values Are Relatively Small: 34 Subjects, Degree 4 Case.

Subject 5

θ	True P.D.F.	Approximation
-3.600	0.00000	0.00000
-3.400	0.00000	0.00000
-3.200	0.00005	0.00005
-3.000	0.00577	0.00582
-2.800	0.11385	0.11477
-2.600	0.70925	0.71495
-2.400	1.74217	1.75618
-2.200	1.75166	1.76575
-2.000	0.62660	0.63164
-1.800	0.05041	0.05082
-1.600	0.00021	0.00021
-1.400	0.00000	0.00000

Subject 94

θ	True P.D.F.	Approximation
1.000	0.00000	0.00000
1.200	0.00000	0.00000
1.400	0.00003	0.00003
1.600	0.00788	0.00792
1.800	0.20173	0.20268
2.000	1.10583	1.11104
2.200	1.95036	1.95955
2.400	1.32600	1.33225
2.600	0.36721	0.36894
2.800	0.03977	0.03995
3.000	0.00146	0.00146
3.200	0.00001	0.00001
3.400	0.00000	0.00000
3.600	0.00000	0.00000
3.800	0.00000	0.00000
4.000	0.00000	0.00000

TABLE A-2-6 (Continued)

Subject 96

θ	True P.D.F.	Approximation
1.600	0.00000	0.00000
1.800	0.03859	0.03916
2.000	0.71529	0.72601
2.200	1.82547	1.85281
2.400	1.66904	1.69405
2.600	0.65370	0.66350
2.800	0.09469	0.09611
3.000	0.00267	0.00271
3.200	0.00000	0.00000

Subject 102

θ	True P.D.F.	Approximation
-2.800	0.45144	0.48232
-2.600	1.25510	1.34095
-2.400	1.84974	1.97627
-2.200	1.61848	1.72919

TABLE A-2-6 (Continued)

Subject 105

θ	True P.D.F.	Approximation
-3.000	0.01345	0.01385
-2.800	0.29127	0.29982
-2.600	1.13630	1.16964
-2.400	1.92157	1.97795
-2.200	1.46016	1.50300
-2.000	0.16297	0.16776

Subject 108

θ	True P.D.F.	Approximation
-4.000	0.00000	0.00000
-3.800	0.00000	0.00000
-3.600	0.00000	0.00000
-3.400	0.00000	0.00000
-3.200	0.00005	0.00005
-3.000	0.00367	0.00368
-2.800	0.07521	0.07556
-2.600	0.54687	0.54944
-2.400	1.58439	1.59185
-2.200	1.86185	1.87062
-2.000	0.81809	0.82195
-1.800	0.10759	0.10809
-1.600	0.00258	0.00259
-1.400	0.00000	0.00000
-1.200	0.00000	0.00000

TABLE A-2-6 (Continued)

Subject 109

θ	True P.D.F.	Approximation
-3.000	0.00833	0.00865
-2.800	0.32279	0.33520
-2.600	1.17297	1.21806
-2.400	1.89740	1.97034
-2.200	1.47531	1.53202
-2.000	0.06316	0.06559

Subject 192

θ	True P.D.F.	Approximation
2.400	1.24160	1.60467
2.600	1.03707	1.34033

TABLE A-2-6 (Continued)

Subject 194

θ	True P.D.F.	Approximation
2.200	1.78075	1.88692
2.400	1.78063	1.88679
2.600	1.04230	1.10444
2.800	0.26137	0.27696

Subject 202

θ	True P.D.F.	Approximation
-3.000	0.01207	0.01243
-2.800	0.30463	0.31440
-2.600	1.16052	1.19774
-2.400	1.92202	1.98366
-2.200	1.44953	1.49602
-2.000	0.12642	0.13047

TABLE A-2-6 (Continued)

<u>Subject</u> 204

θ	True P.D.F.	Approximation
-3.200	0.00000	0.00000
-3.000	0.01271	0.01300
-2.800	0.24985	0.25556
-2.600	1.07014	1.09459
-2.400	1.91675	1.96055
-2.200	1.49268	1.52679
-2.000	0.25896	0.26488

<u>Subject</u> 206

θ	True P.D.F.	Approximation
-3.200	0.00001	0.00001
-3.000	0.00923	0.00938
-2.800	0.19246	0.19574
-2.600	0.94803	0.96417
-2.400	1.87027	1.90212
-2.200	1.57916	1.60605
-2.000	0.39868	0.40547
-1.800	0.00312	0.00317

TABLE A-2-6 (Continued)

<u>Subject</u> 209

θ	True P.D.F.	Approximation
-3.600	0.00000	0.00000
-3.400	0.00000	0.00000
-3.200	0.00002	0.00002
-3.000	0.00305	0.00307
-2.800	0.07539	0.07590
-2.600	0.55968	0.56350
-2.400	1.58986	1.60072
-2.200	1.85056	1.86320
-2.000	0.81702	0.82259
-1.800	0.10279	0.10349
-1.600	0.00171	0.00172
-1.400	0.00000	0.00000

<u>Subject</u> 210

θ	True P.D.F.	Approximation
-3.800	0.00000	0.00000
-3.600	0.00000	0.00000
-3.400	0.00000	0.00000
-3.200	0.00002	0.00002
-3.000	0.00283	0.00284
-2.800	0.06763	0.06803
-2.600	0.51709	0.52021
-2.400	1.53716	1.54646
-2.200	1.87525	1.88659
-2.000	0.87496	0.88025
-1.800	0.12236	0.12310
-1.600	0.00279	0.00281
-1.400	0.00000	0.00000
-1.200	0.00000	0.00000

TABLE A-2-6 (Continued)

<u>Subject</u> 288

θ	True P.D.F.	Approximation
1.000	0.00000	0.00000
1.200	0.00000	0.00000
1.400	0.00010	0.00010
1.600	0.01621	0.01629
1.800	0.29110	0.29252
2.000	1.28009	1.28636
2.200	1.93750	1.94699
2.400	1.16404	1.16974
2.600	0.28425	0.28564
2.800	0.02608	0.02621
3.000	0.00073	0.00073
3.200	0.00000	0.00000
3.400	0.00000	0.00000
3.600	0.00000	0.00000
3.800	0.00000	0.00000

<u>Subject</u> 293

θ	True P.D.F.	Approximation
1.800	0.03637	0.03692
2.000	0.70129	0.71179
2.200	1.81746	1.84468
2.400	1.67928	1.70443
2.600	0.66460	0.67455
2.800	0.09765	0.09911
3.000	0.00283	0.00287
3.200	0.00000	0.00000

TABLE A-2-6 (Continued)

Subject 294

θ	True P.D.F.	Approximation
1.800	0.00004	0.00004
2.000	0.52788	0.54042
2.200	1.74699	1.78849
2.400	1.77188	1.81397
2.600	0.81440	0.83375
2.800	0.14293	0.14633
3.000	0.00323	0.00331

Subject 302

θ	True P.D.F.	Approximation
-3.000	0.00995	0.01035
-2.800	0.35045	0.36482
-2.600	1.20945	1.25903
-2.400	1.91319	1.99162
-2.200	1.45075	1.51023

TABLE A-2-6 (Continued)

Subject 303

θ	True P.D.F.	Approxi-mation
-3.800	0.00000	0.00000
-3.600	0.00000	0.00000
-3.400	0.00000	0.00000
-3.200	0.00003	0.00003
-3.000	0.00307	0.00308
-2.800	0.06889	0.06926
-2.600	0.52049	0.52332
-2.400	1.54681	1.55524
-2.200	1.87631	1.88653
-2.000	0.86234	0.86704
-1.800	0.11922	0.11987
-1.600	0.00288	0.00289
-1.400	0.00000	0.00000
-1.200	0.00000	0.00000

Subject 305

θ	True P.D.F.	Approxi-mation
-3.800	0.00000	0.00000
-3.600	0.00000	0.00000
-3.400	0.00000	0.00000
-3.200	0.00006	0.00006
-3.000	0.00535	0.00538
-2.800	0.10287	0.10359
-2.600	0.66157	0.66618
-2.400	1.69908	1.71091
-2.200	1.78793	1.80039
-2.000	0.67927	0.68400
-1.800	0.06337	0.06382
-1.600	0.00049	0.00049
-1.400	0.00000	0.00000

TABLE A-2-6 (Continued)

Subject 308

θ	True P.D.F.	Approxi-mation
-3.600	0.00000	0.00000
-3.400	0.00000	0.00000
-3.200	0.00002	0.00003
-3.000	0.00398	0.00401
-2.800	0.09264	0.09339
-2.600	0.63234	0.63749
-2.400	1.66296	1.67652
-2.200	1.80464	1.81935
-2.000	0.72807	0.73401
-1.800	0.07463	0.07524
-1.600	0.00061	0.00061
-1.400	0.00000	0.00000

Subject 392

θ	True P.D.F.	Approxi-mation
1.600	0.00009	0.00010
1.800	0.06382	0.06448
2.000	0.76473	0.77261
2.200	1.[illegible]10	1.87927
2.400	1.62962	1.64642
2.600	0.59455	0.60067
2.800	0.08366	0.08453
3.000	0.00324	0.00327
3.200	0.00001	0.00001
3.400	0.00000	0.00000

TABLE A-2-6 (Continued)

<u>Subject</u> 395

θ	True P.D.F.	Approximation
1.800	0.01059	0.01077
2.000	0.58571	0.59604
2.200	1.77061	1.80185
2.400	1.75354	1.78447
2.600	0.75255	0.76583
2.800	0.12343	0.12561
3.000	0.00392	0.00399
3.200	0.00000	0.00000

<u>Subject</u> 396

θ	True P.D.F.	Approximation
1.200	0.00000	0.00000
1.400	0.00002	0.00002
1.600	0.00875	0.00880
1.800	0.21949	0.22069
2.000	1.14798	1.15423
2.200	1.94782	1.95843
2.400	1.28974	1.29677
2.600	0.34886	0.35076
2.800	0.03618	0.03637
3.000	0.00119	0.00119
3.200	0.00001	0.00001
3.400	0.00000	0.00000
3.600	0.00000	0.00000
3.800	0.00000	0.00000

TABLE A-2-6 (Continued)

Subject 397

θ	True P.D.F.	Approxi-mation
2.400	0.62053	1.14678
2.600	0.64313	1.18856

Subject 398

θ	True P.D.F.	Approxi-mation
2.200	1.79245	1.90437
2.400	1.75832	1.86807
2.600	1.03909	1.10395
2.800	0.26027	0.27651

TABLE A-2-6 (Continued)

Subject 400

θ	True P.D.F.	Approximation
1.200	0.00000	0.00000
1.400	0.00002	0.00002
1.600	0.00986	0.00992
1.800	0.24014	0.24160
2.000	1.19322	1.20043
2.200	1.94335	1.95510
2.400	1.25063	1.25819
2.600	0.32940	0.33140
2.800	0.03252	0.03271
3.000	0.00094	0.00095
3.200	0.00000	0.00000
3.400	0.00000	0.00000
3.600	0.00000	0.00000

Subject 404

θ	True P.D.F.	Approximation
-3.400	0.00000	0.00000
-3.200	0.00003	0.00003
-3.000	0.00592	0.00598
-2.800	0.12358	0.12484
-2.600	0.74439	0.75200
-2.400	1.75978	1.77778
-2.200	1.72967	1.74737
-2.000	0.59771	0.60382
-1.800	0.03883	0.03923
-1.600	0.00003	0.00003

TABLE A-2-6 (Continued)

Subject 406

θ	True P.D.F.	Approximation
-3.800	0.00000	0.00000
-3.600	0.00000	0.00000
-3.400	0.00000	0.00000
-3.200	0.00006	0.00006
-3.000	0.00541	0.00545
-2.800	0.10373	0.10445
-2.600	0.66478	0.66941
-2.400	1.70207	1.71393
-2.200	1.78528	1.79772
-2.000	0.67553	0.68024
-1.800	0.06264	0.06308
-1.600	0.00048	0.00048
-1.400	0.00000	0.00000

Subject 409

θ	True P.D.F.	Approximation
-3.800	0.00000	0.00000
-3.600	0.00000	0.00000
-3.400	0.00000	0.00000
-3.200	0.00003	0.00003
-3.000	0.00319	0.00321
-2.800	0.07074	0.07112
-2.600	0.52895	0.53183
-2.400	1.55723	1.56571
-2.200	1.87091	1.88110
-2.000	0.85041	0.85504
-1.800	0.11585	0.11649
-1.600	0.00273	0.00275
-1.400	0.00000	0.00000
-1.200	0.00000	0.00000

TABLE A-2-6 (Continued)

Subject 494

θ	True P.D.F.	Approxi-mation
1.600	0.00009	0.00009
1.800	0.06323	0.06388
2.000	0.76181	0.76966
2.200	1.85856	1.87771
2.400	1.63196	1.64878
2.600	0.59671	0.60286
2.800	0.08419	0.08506
3.000	0.00327	0.00330
3.200	0.00001	0.00001
3.400	0.00000	0.00000

Subject 495

θ	True P.D.F.	Approxi-mation
1.200	0.00000	0.00000
1.400	0.00000	0.00000
1.600	0.00463	0.00465
1.800	0.16766	0.16864
2.000	1.03625	1.04230
2.200	1.94149	1.95281
2.400	1.39242	1.40054
2.600	0.40912	0.41151
2.800	0.04680	0.04707
3.000	0.00173	0.00174
3.200	0.00001	0.00001
3.400	0.00000	0.00000
3.600	0.00000	0.00000
3.800	0.00000	0.00000

TABLE A-2-6 (Continued)

Subject 498

θ	True P.D.F.	Approximation
2.000	0.26702	0.27827
2.200	1.71724	1.78959
2.400	1.80033	1.87618
2.600	0.97042	1.01130
2.800	0.20798	0.21675
3.000	0.00022	0.00023

Subject 499

θ	True P.D.F.	Approximation
2.000	0.35865	0.37244
2.200	1.71788	1.78392
2.400	1.79140	1.86026
2.600	0.93857	0.97465
2.800	0.19069	0.19802
3.000	0.00053	0.00055

TABLE A-2-7

Comparison of Pearson's Type II Density Function with the One Approximated by Stirling's Formula When the Parameter Values Are Relatively Small: 2 Subjects, Degree 4 Case.

Subject 301

θ	True P.D.F.	Approximation
-3.800	0.00000	0.00000
-3.600	0.00000	0.00000
-3.400	0.00000	0.00000
-3.200	0.00001	0.00001
-3.000	0.00084	0.00084
-2.800	0.02826	0.02839
-2.600	0.29718	0.29855
-2.400	1.18233	1.18780
-2.200	1.92239	1.93127
-2.000	1.25725	1.26306
-1.800	0.29333	0.29469
-1.600	0.01842	0.01850
-1.400	0.00017	0.00017
-1.200	0.00000	0.00000
-1.000	0.00000	0.00000

Subject 405

θ	True P.D.F.	Approximation
-3.800	0.00000	0.00000
-3.600	0.00000	0.00000
-3.400	0.00000	0.00000
-3.200	0.00001	0.00001
-3.000	0.00084	0.00084
-2.800	0.02822	0.02835
-2.600	0.29691	0.29828
-2.400	1.18179	1.18724
-2.200	1.92235	1.93122
-2.000	1.25783	1.26364
-1.800	0.29364	0.29499
-1.600	0.01845	0.01854
-1.400	0.00017	0.00017
-1.200	0.00000	0.00000
-1.000	0.00000	0.00000

TABLE A-2-8

Discrimination Parameter and Its Estimates of Each of the Ten Binary Items Obtained by the Pearson System Method, with the Estimate Obtained from the Criterion Item Characteristic Function, Using Each of the Four Different Ranges of the Estimated Item Characteristic Functions, [0.15, 0.85], [0.10, 0.90], [0.05, 0.95] and [0.01, 0.99], within the Interval of θ , [-2.4, 2.4].

METHOD		DGR. 3				DGR. 4				CRITERION			
ITEM	TRUE a_g	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99
1	1.5	0.5856_3	0.8095_4	1.1773_5	1.5298	0.3153_3	0.6634_4	1.0896_5	1.3342	0.9536_3	1.1064_4	1.4003_5	1.5517
2	1.0	0.9006	1.0402	1.0535	1.0825	0.9931	1.0762	1.0969	1.1112	0.8595	0.9693	1.0238	1.0583
3	2.5	1.9671	1.9671	1.7921	1.9033	1.9720	1.9720	1.8381	1.8770	1.9510	1.8923	1.7876	1.8960
4	1.0	0.8327	0.8251	0.8829	0.9108	0.8262	0.8180	0.8803	0.9059	0.8181	0.8114	0.8679	0.8952
5	1.5	1.3244	1.3583	1.3823	1.4526	1.3034	1.3425	1.3657	1.4395	1.3093	1.3464	1.3682	1.4376
6	1.0	0.8703	0.8672	0.9054	0.8631	0.8623	0.8581	0.8966	0.8860	0.8415	0.8585	0.8950	0.8631
7	2.0	1.5755	1.5295	1.4844	1.4964	1.5724	1.5239	1.4753	1.4860	1.5710	1.5214	1.4725	1.4814
8	1.0	0.8291	0.8460	0.8867	0.9820	0.8278	0.8446	0.9047	0.9673	0.8248	0.8460	0.8860	0.9906
9	2.0	1.8009_5	1.7395	1.6730	1.7086	1.8365_5	1.7474	1.6527	1.6935	1.8275_5	1.7542	1.7165	1.7445
10	1.0	0.6964	0.6940	0.7141	0.8753	0.6834	0.6845	0.7068	0.8717	0.7128	0.7076	0.7255	0.8839

The number of intervals used in estimation is shown as a subscript when it is less than 6 .

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TABLE A-2-9

Difficulty Parameter and Its Estimates of Each of the Ten Binary Items Obtained by the Pearson System Method, with the Estimate Obtained from the Criterion Item Characteristics Function, Using Each of the Four Different Ranges of the Estimated Item Characteristic Functions, [0.15, 0.85], [0.10, 0.90], [0.05, 0.95] and [0.01, 0.99], within the Interval of θ , [-2.4, 2.4].

METHOD		DGR. 3				DGR. 4				CRITERION			
ITEM	TRUE b_g	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99	0.15-0.85	0.10-0.90	0.05-0.95	0.01-0.99
1	-2.5	-3.3556_3	-3.0545_4	-2.7875_5	-2.6309	-4.4909_3	-3.3236_4	-2.8841_5	-2.7465	-2.8510_3	-2.7703_4	-2.6506_5	-2.6001
2	-2.0	-1.9651	-1.9677	-1.9664	-1.9609	-1.9380	-1.9428	-1.9424	-1.9400	-2.0081	-2.0072	-2.0022	-1.9943
3	-1.5	-1.4762	-1.4762	-1.4939	-1.5134	-1.4857	-1.4857	-1.4981	-1.5057	-1.4896	-1.4945	-1.5068	-1.5252
4	-1.0	-1.0446	-1.0425	-0.9898	-1.0004	-1.0523	-1.0501	-0.9910	-1.0007	-1.0589	-1.0572	-1.0048	-1.0148
5	-0.5	-0.4840	-0.4696	-0.4681	-0.4785	-0.4901	-0.4741	-0.4719	-0.4796	-0.4885	-0.4732	-0.4719	-0.4817
6	0.0	-0.0460	-0.0644	-0.0727	-0.0416	-0.0441	-0.0632	-0.0734	-0.0739	-0.0531	-0.0640	-0.0747	-0.0354
7	0.5	0.4960	0.5208	0.5215	0.5133	0.5019	0.5263	0.5266	0.5176	0.5000	0.5251	0.5268	0.5191
8	1.0	0.9674	0.9614	0.9387	0.9856	0.9736	0.9655	0.9673	0.9953	0.9745	0.9645	0.9812	0.9779
9	1.5	1.4820_5	1.5014	1.5019	1.5081	1.4926_5	1.5084	1.5155	1.5224	1.4963_5	1.5088	1.5016	1.5064
10	2.0	2.1366	2.1375	2.1280	2.0472	2.1601	2.1596	2.1483	2.0616	2.1237	2.1256	2.1176	2.0408

The number of intervals used in estimation is shown as a subscript when it is less than 6 .

APPENDIX III

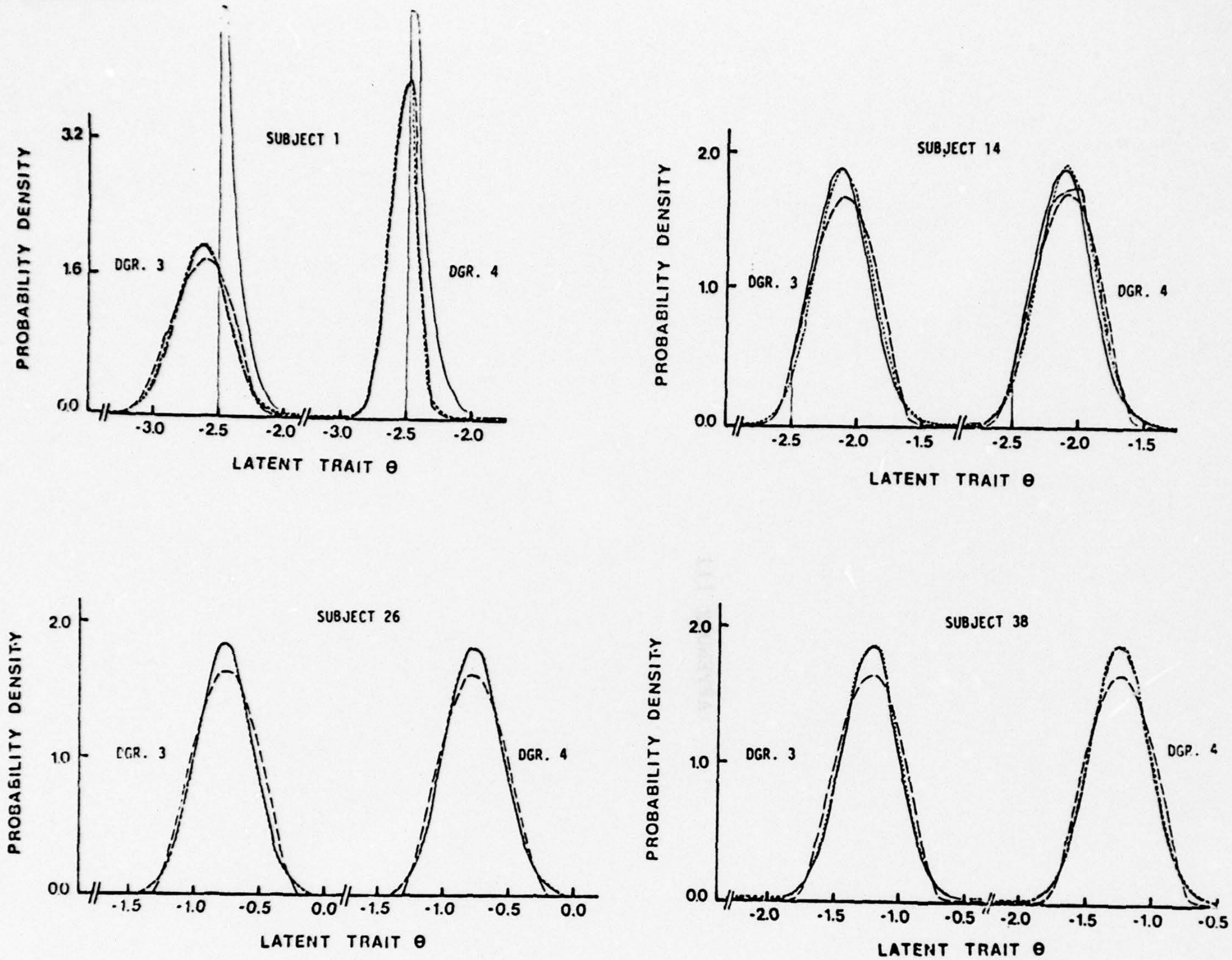


FIGURE A-3-1

The conditional probability density of θ , given $\hat{\theta}$ (Solid Curve), and its estimates by the Normal Approach Method (Dashed Curve) and by the Two-Parameter Beta Method (Broken Curve). The estimated density by the Pearson System Method is often the same as that obtained by the Normal Approach Method; if not, it is drawn by a broken and dotted curve.

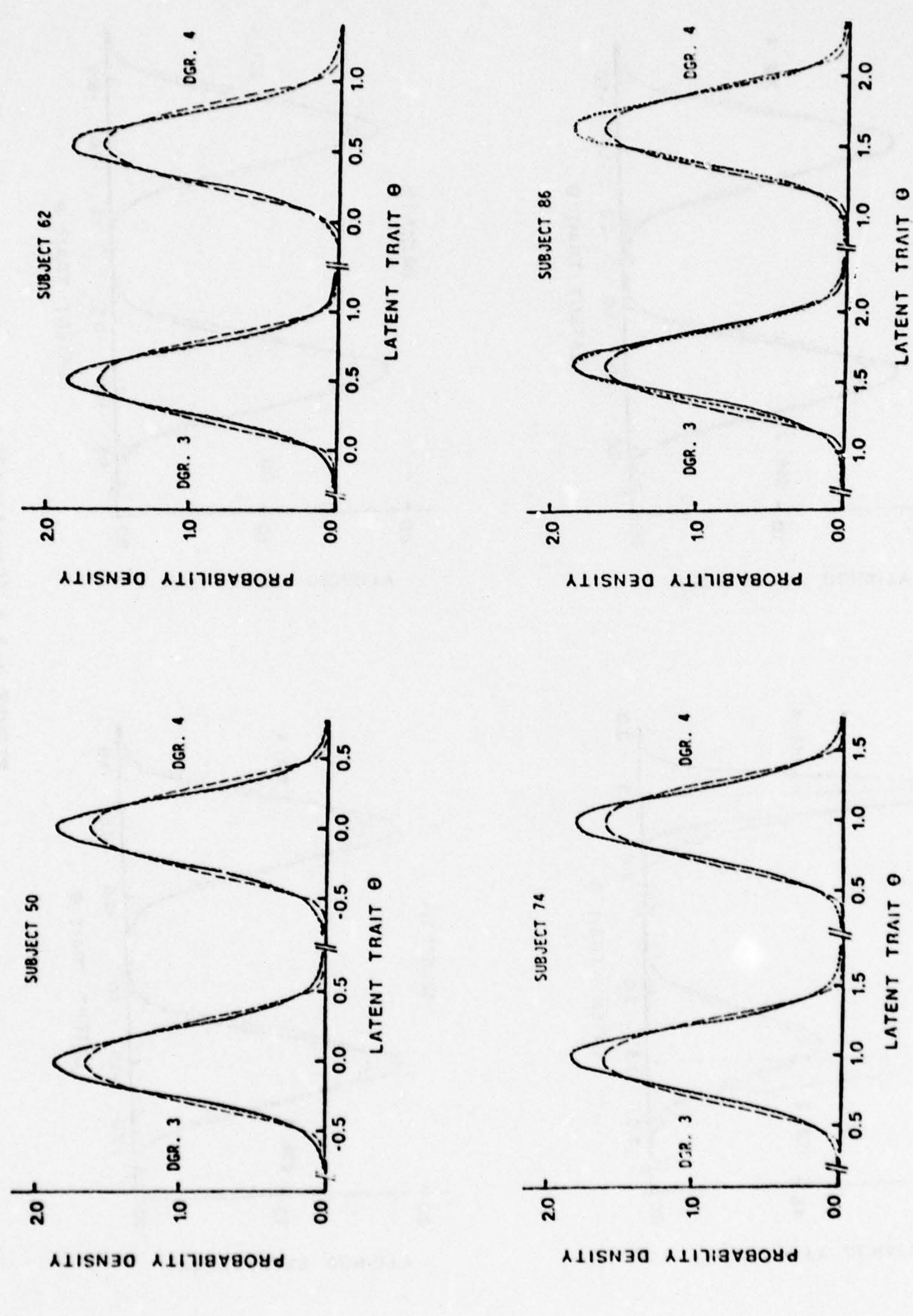
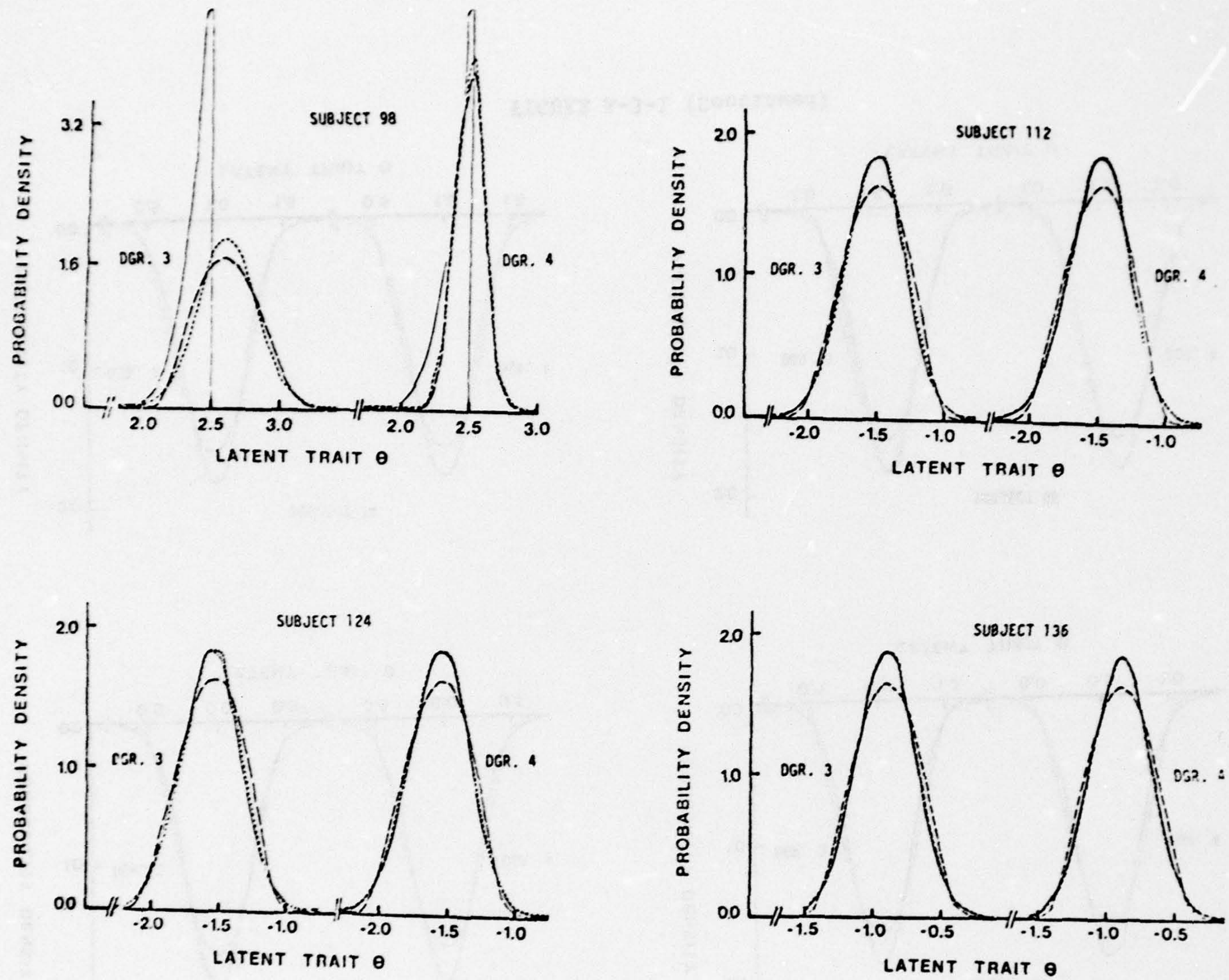


FIGURE A-3-1 (Continued)



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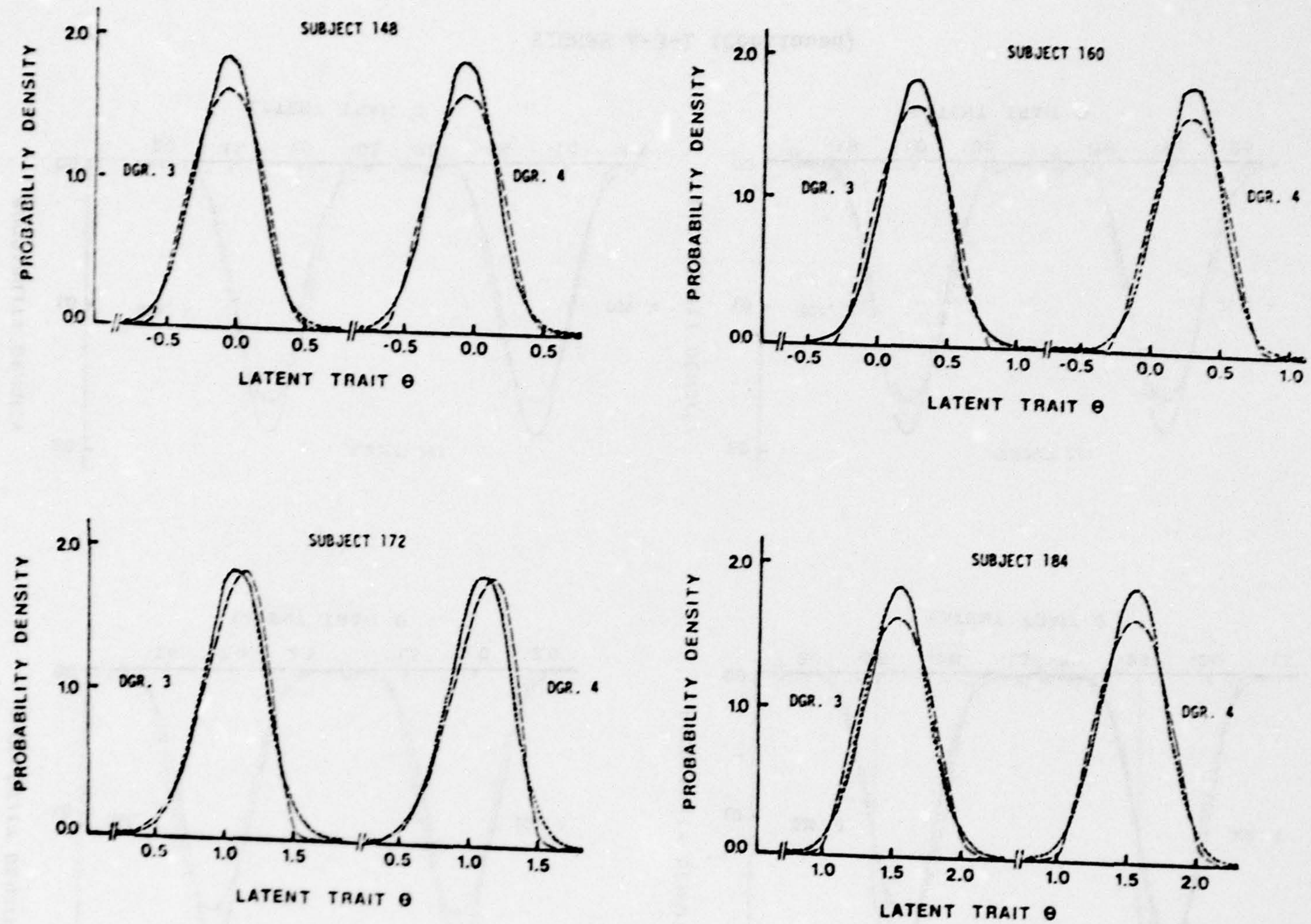

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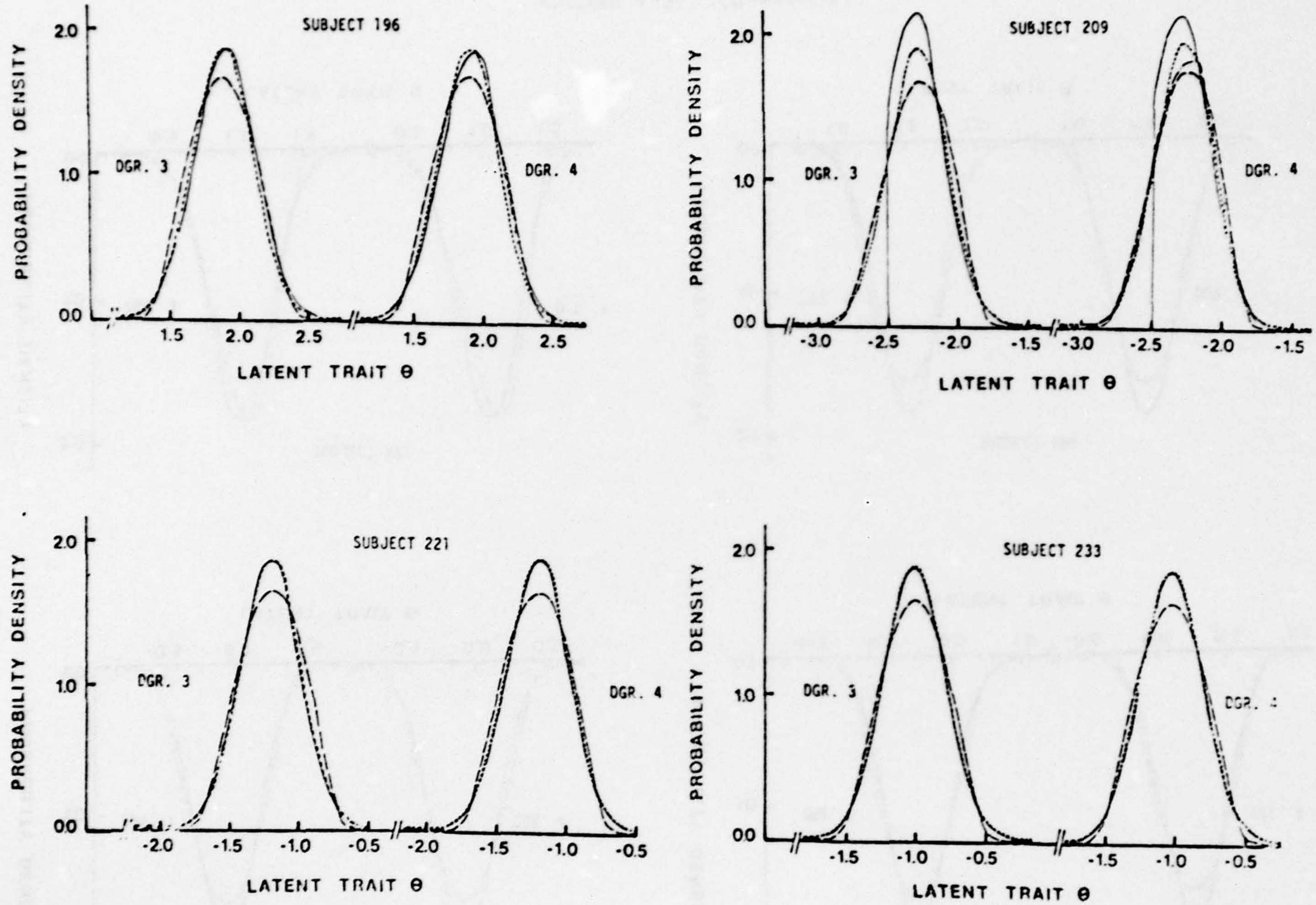

FIGURE A-3-1 (Continued)

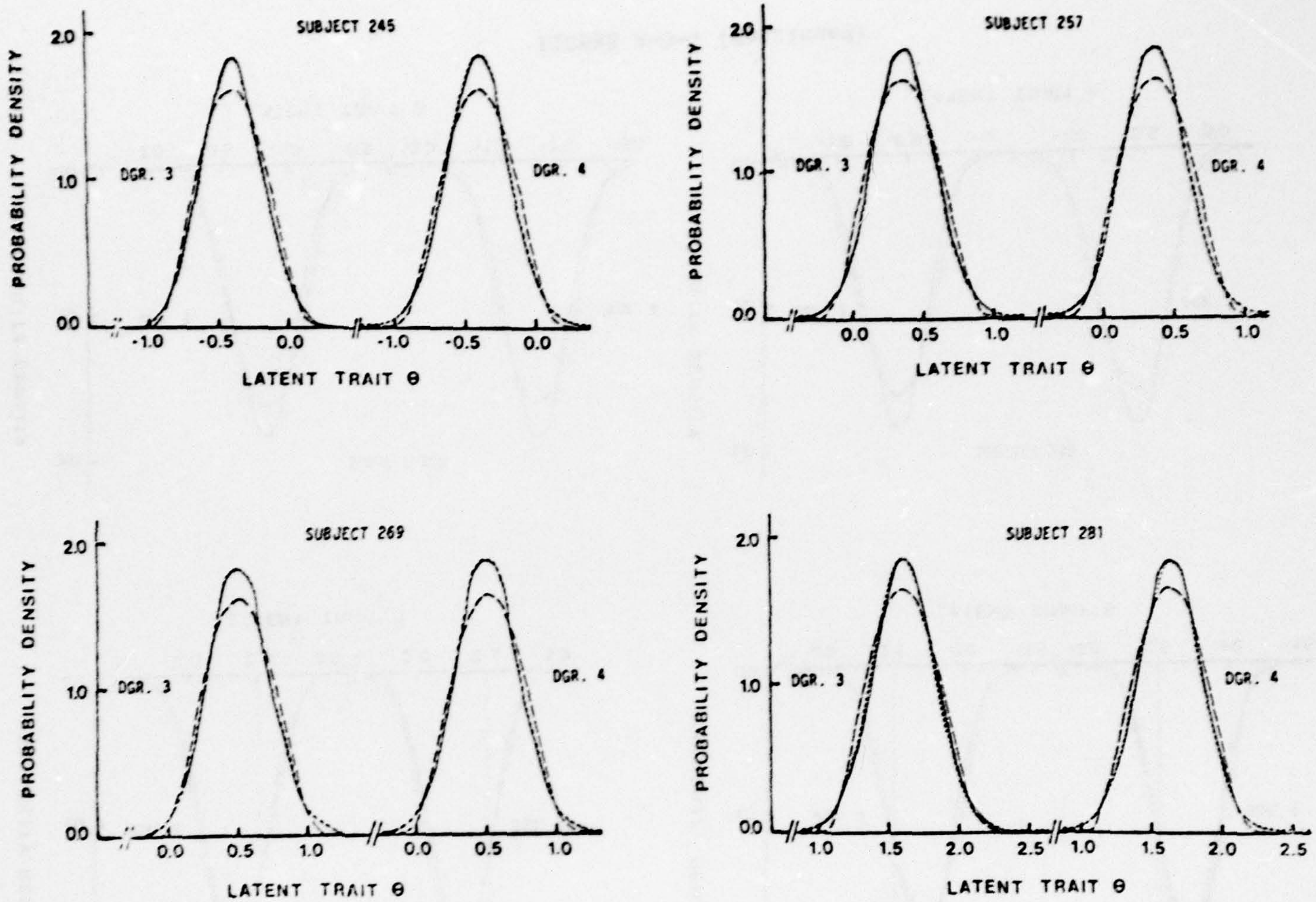

FIGURE A-3-1 (Continued)

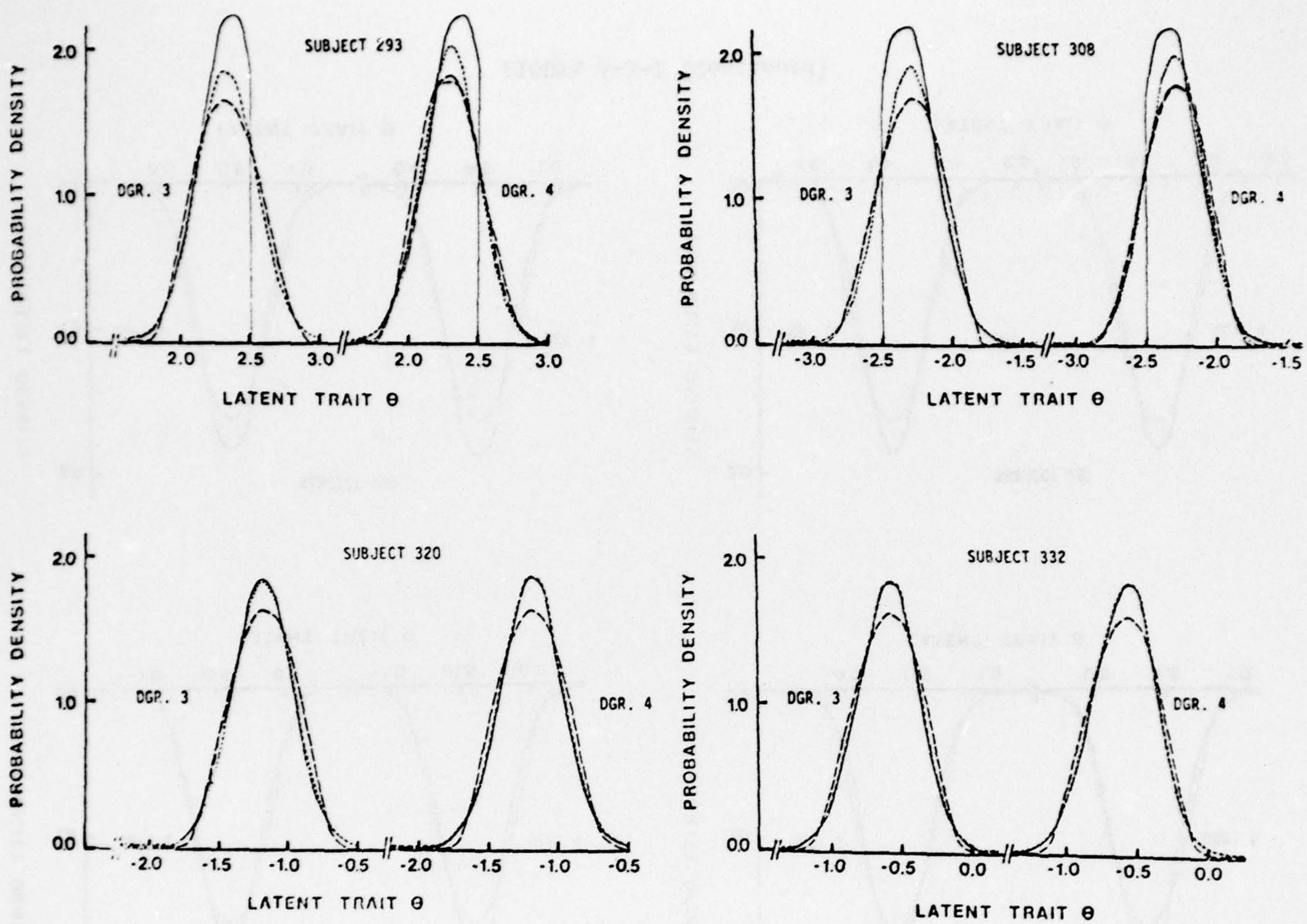

FIGURE A-3-1 (Continued)

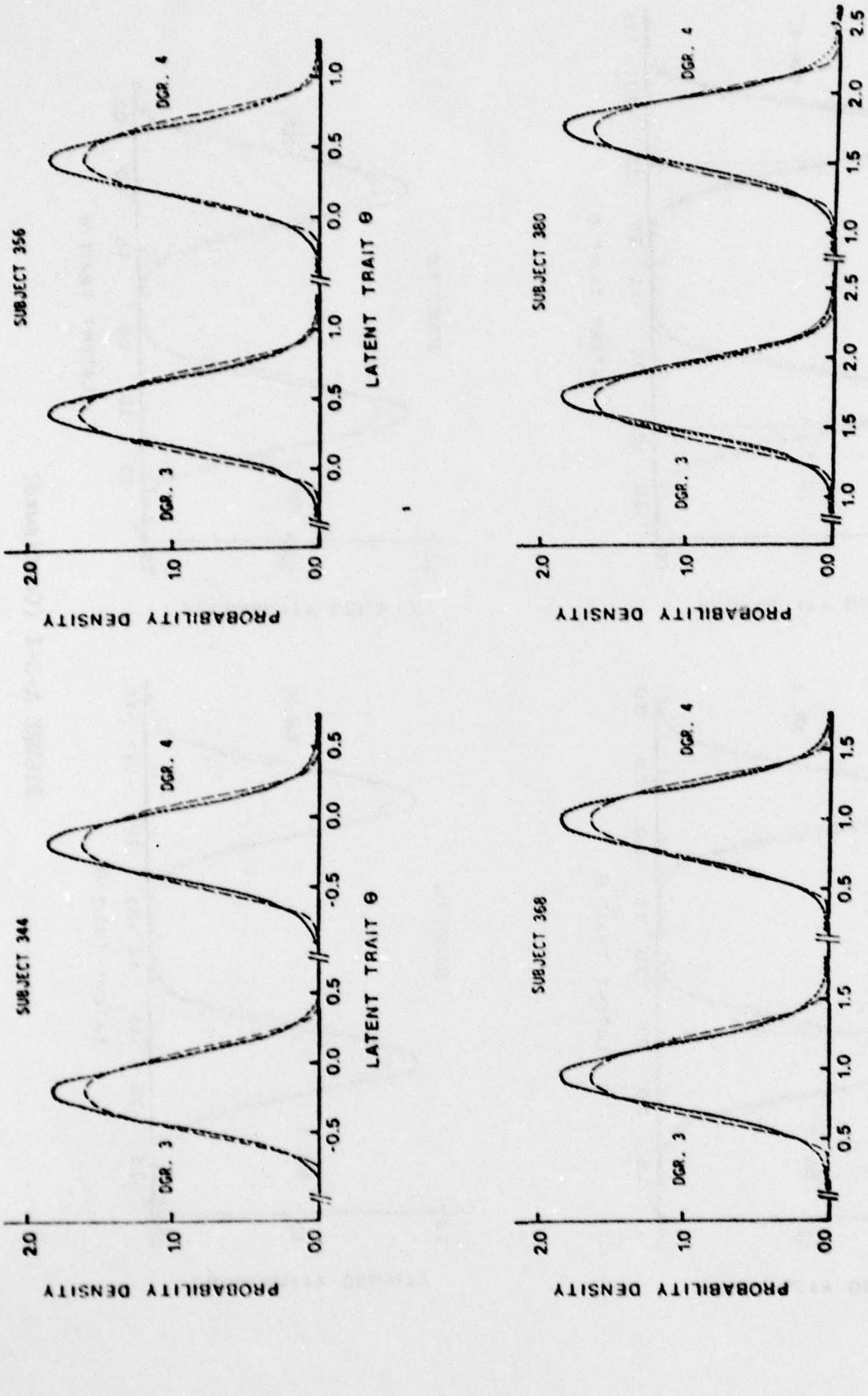

FIGURE A-3-1 (Continued)

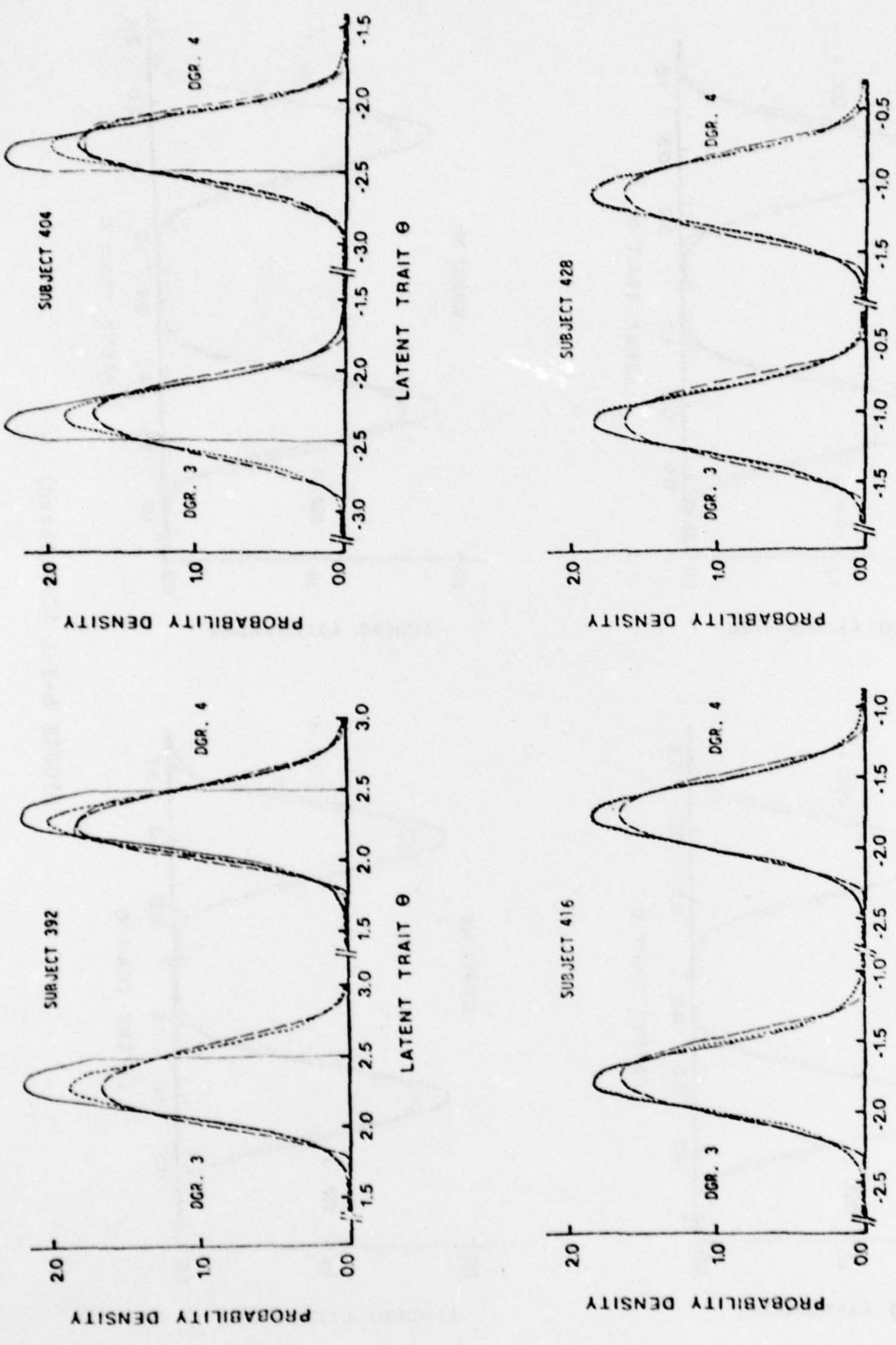

FIGURE A-3-1 (Continued)

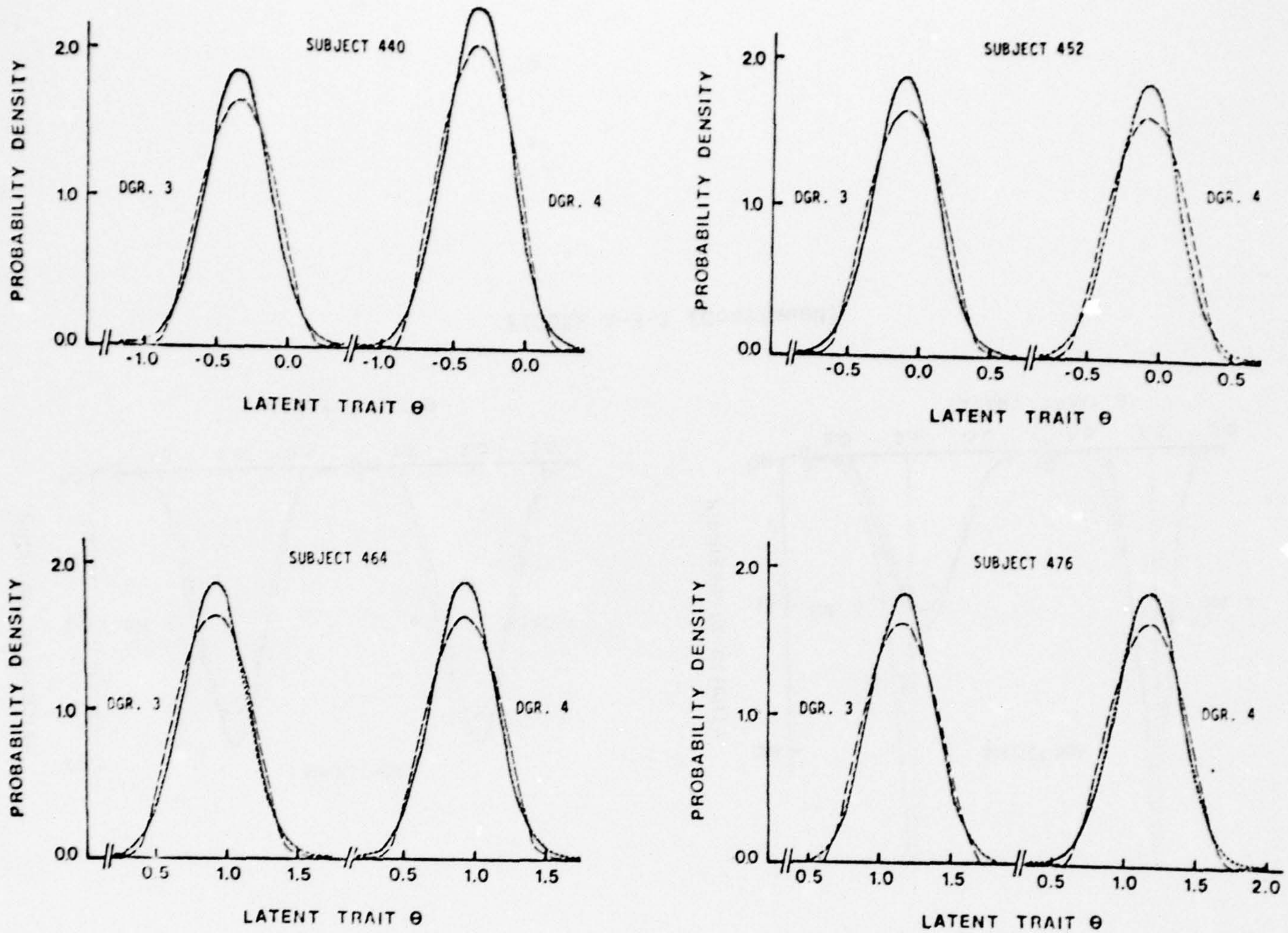

FIGURE A-3-1 (Continued)

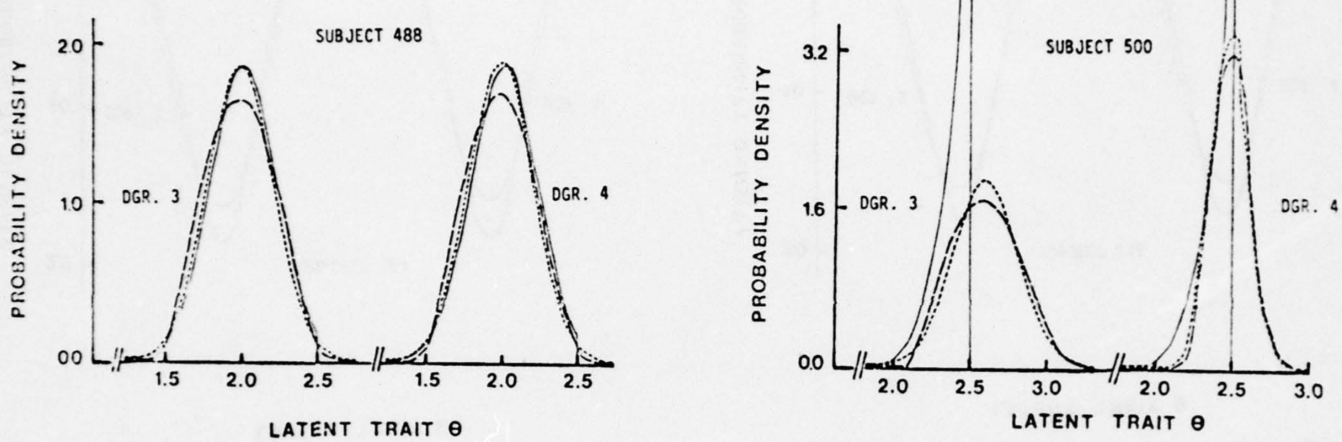

FIGURE A-3-1 (Continued)

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